Robotics
Time sensors

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Wednesday 14th December, 2016

http://robot.unipv.it/toolleeo

time is a fundamental parameter, which must be accounted in almost every application

the measurement of time is done in a computing system for several purposes:

- to associate a timestamp to an event
- to execute an operation at a given time instant
- to determine the duration of an event
- to keep the time ordering of a sequence of events

the time sensor is based on the so-called **clock** in the 13rd conference on weights and measures (1967), the second has been defined as

> the duration of 9.192.631.770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom

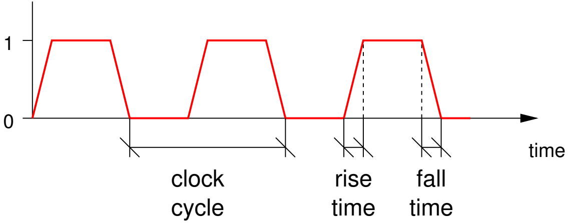
in 1997 the definition has been made more accurate by stating that

> refers to a caesium atom at rest at a temperature of 0 K

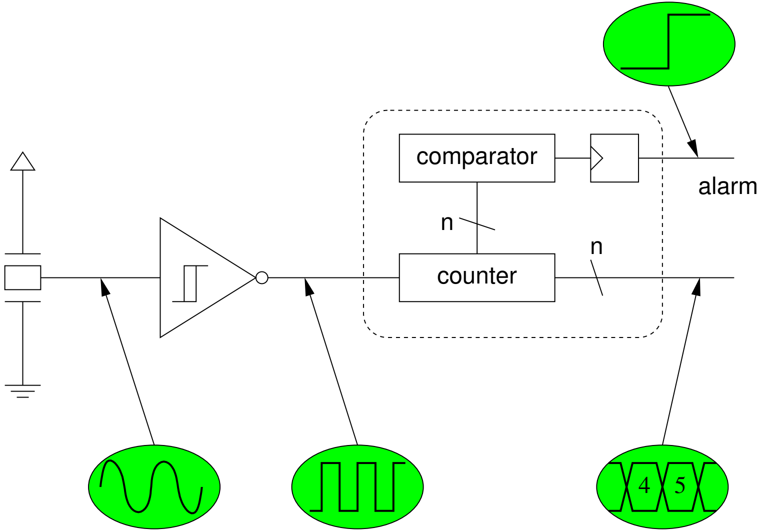
The oscillator

in electronics, the time is measured by counting the number of state changes of an oscillator having known frequency

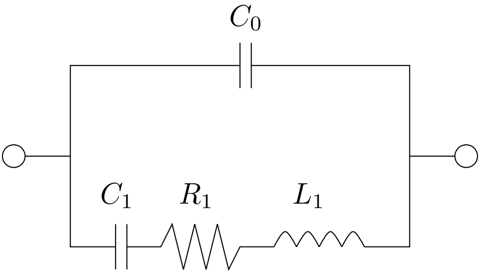
a typical clock signal has the following features:



Circuit that generates the clock



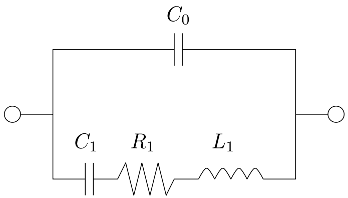
a quartz oscillator has the following equivalent circuit:



its impedence is

$$Z(s) = \left(\frac{1}{sC_1} + sL_1 + R_1\right) \parallel \left(\frac{1}{sC_0}\right)$$

Equivalent circuit



- R_1, C_1 and L_1 depend from the characteristics of the crystal
- C_0 is the capacity formed by the two faces of the crystal where probes are applied (usually $C_0 \gg C_1$)
- the oscillator is characterized by two resonance frequencies, namely the series (ω_s) and parallel (ω_p) frequency

$$\omega_s = \sqrt{\frac{1}{L_1 C_1}} \qquad \omega_p = \sqrt{\frac{C_1 + C_0}{L_1 C_1 C_0}}$$

Types of oscillators

ATCXO: analog temperature controlled c.o.

- the profile of variation of the frequency as a function of the temperature is measured
- a compensation table is implemented in silicon into the component

OCVCXO: oven-controlled temperature compensated c.o.

- the crystal is located within a temperature-compensated room
- the room is insulated and thermo-resistors are used to maintain the desired temperature
- very good performance, but required extra input power

VCTCXO: voltage controlled voltage-controlled c.o.

- the frequency is stabilized by controlling the supply voltage of the crystal
- the voltage can also (but by a limited extent) compensate the effect of the temperature

Types of oscillators

ATCXO	analog temperature controlled c.o. (crystal oscillator)
CDXO	calibrated dual c.o.
MCXO	microcomputer-compensated c.o.
OCVCXO	oven-controlled voltage-controlled c.o.
OCXO	oven-controlled c.o.
RbXO	rubidium c.o. (RbXO)
TCVCXO	temperature-compensated voltage-controlled c.o.
TCXO	temperature-compensated c.o.
TSXO	temperature-sensing c.o., an adaptation of the TCXO
VCTCXO	voltage controlled temperature compensated c.o.
VCXO	voltage-controlled c.o.
DTCXO	digital temperature compensated c.o.

Characteristics of oscillators

	TCXO	MCXO	OCXO	RbXO	caesium
Aging/year	$5 \cdot 10^{-7}$	$2 \cdot 10^{-8}$	$5 \cdot 10^{-9}$	$2 \cdot 10^{-10}$	0
Size (cm^2)	10	50	20-200	1200	6000
Warmup time (min.)	0.1 $1 \cdot 10^{-6}$	0.1 $2 \cdot 10^{-8}$	4 $1 \cdot 10^{-8}$	3 $5 \cdot 10^{-10}$	20 $2 \cdot 10^{-11}$
Power (W)	0.05	0.04	0.6	0.65	30
Price ($\sim$\$)	100	1000	2000	10000	40000

- aging may be confused with the drift
- more precisely:
  - the aging is a parameter of an oscillator
  - the drift depends on the application, being affected by the aging and other factors

The clock drift

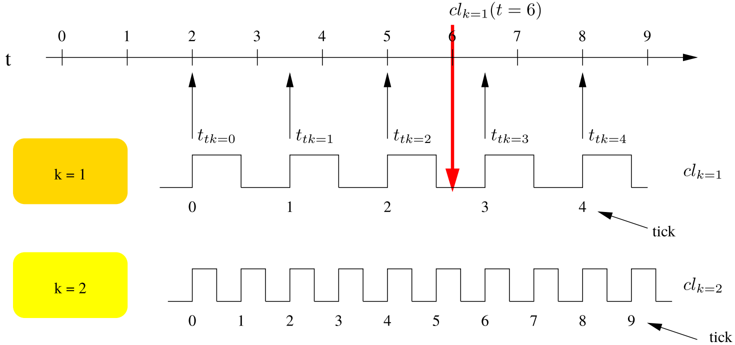
in many applications (e.g., distributed systems) it is mandatory for the involved machines to share the same value of the time

due to the limited accuracy of oscillators, in few time the clock generated by the oscillators tends to drift from the "true" time

this is the so-called **clock drift**

- oscillators having different characteristics have different drifts as well
- oscillators having similar characteristics have slightly different drifts anyway
- the temperature plays an important role in the clock drift

True time and local clocks



- t is the true time
- cl_k is the physical local clock of the device k
- $cl_k(t)$ indicates the value of cl_k at time t
- t_{tk} is the true time value associated to the tk-th system tick ($tk = 0, 1, 2, \ldots$)

the clock granularity g is defined as

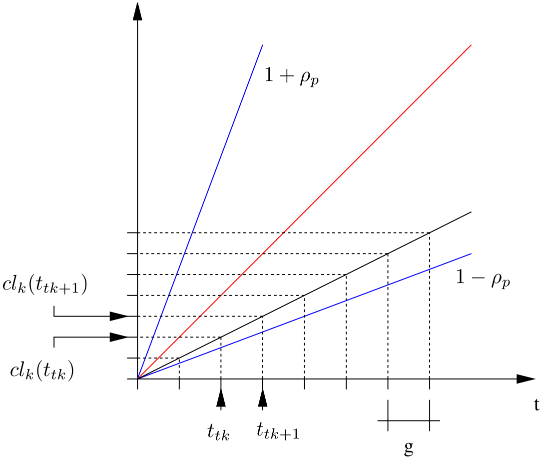
$$t_{tk+1} - t_{tk} = g$$

to be of any practical use, a clock must have a bounded drift; i.e., a value ρ_p must exist such as

$$0 \leq 1 - \rho_p \leq \frac{cl_k(t_{tk+1}) - cl_k(t_{tk})}{g} \leq 1 + \rho_p$$

The clock drift

$$0 \leq 1 - \rho_p \leq \frac{cl_k(t_{tk+1}) - cl_k(t_{tk})}{g} \leq 1 + \rho_p$$

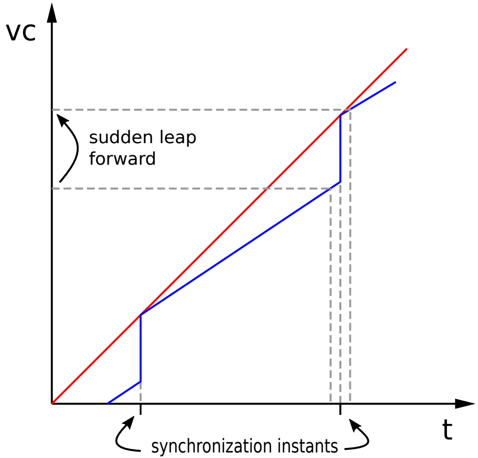


global clock (virtual clock)

it is the global vision of the time as shared by all the machines composing the distributed system

- in the initial condition, the virtual clock vc_k of each node k is close to a global value
- since the clocks constantly drift away, they must be periodically synchronized

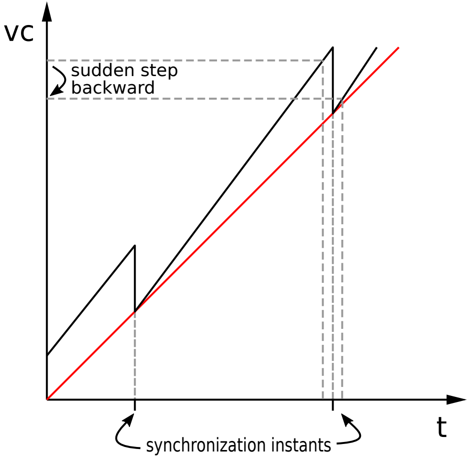
the virtual clock of one machine is periodically re-set to be close to a reference clock



PROBLEM: there are discontinuities in the function that describes the virtual clock

- the sudden leap forward may lead to a deadline violation
- the sudden step backward may lead to wrong ordering of events

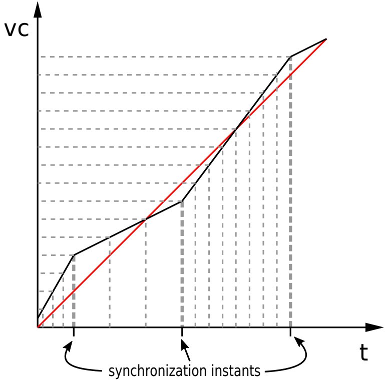
the virtual clock of one machine is periodically re-set to be close to a reference clock



PROBLEM: there are discontinuities in the function that describes the virtual clock

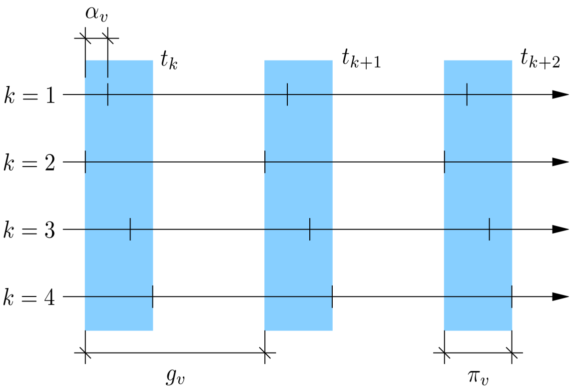
- the sudden leap forward may lead to a deadline violation
- the sudden step backward may lead to wrong ordering of events

there is a periodic update of the mapping between the virtual clock and the associated physical clock durations



the logical duration of one tick or the virtual clock changes between different synchronization instants

Clock synchronization



- g_v : granularity
- α_v : accuracy
- π_v : precision

the convergence δ_v indicates how much the virtual clocks of 2 machines k and l are close each other just after the synchronization event $tk = sync$:

$$|t(vc_k^{tk=sync}) - t(vc_l^{tk=sync})| \leq \delta_v$$

the precision π_v indicates how much two clocks are close each others at any time instant:

$$\forall tk \geq 0 : |t(vc_k^{tk}) - t(vc_l^{tk})| \leq \pi_v$$

- the precision can not be better than the convergence
- it depends from the drift of local clocks
- it is user-defined: can be balanced with the overhead due to the necessary synchronizations

Parameters and properties

the drift rate ρ_v indicates the instantaneous drift occurring at two consecutive ticks, i.e., for $0 \leq t_{tk} \leq t_{tk+1}$

$$0 \leq 1 - \rho_v \leq \frac{vc_k(t_{tk+1}) - vc_k(t_{tk})}{g} \leq 1 + \rho_v$$

the envelope rate ρ_α indicates the long-period drift, i.e., for $t \geq 0$

$$0 \leq 1 - \rho_\alpha \leq \frac{vc_k(t) - vc_k(0)}{t} \leq 1 + \rho_\alpha$$

the accuracy α_v indicates how much a virtual clock is close to an external reference of true time, i.e., for $\forall tk$

$$|t(vc_k^{tk}) - t_{tk}| \leq \alpha_v$$

Accuracy and external synchronization

- the concept of accuracy makes sense when one or more machines need to be synchronized with an external clock source
- the use of the GPS is an effective technique for centralized synchronization ($\alpha_g \leq 100$ ns)
- the GPS is a typical example of external clock source to which a machine may wish to be synchronized

PROBLEM
the GPS signal is not available everywhere

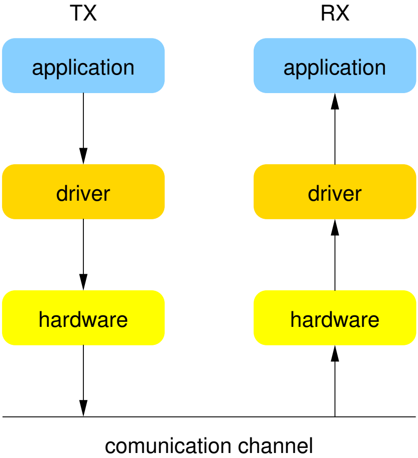
- a master node periodically sends a synchronization message to all client nodes
- the message contains a timestamp of the master clock
- all nodes get synchronized with the master clock

time critical path

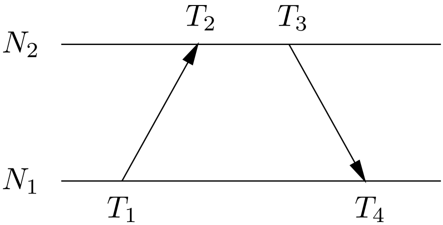
time is required for the following operations:

1. acquisition of the timestamp by the server
2. send the message containing the timestamp
3. reception of the message by the client
4. application of the clock synchronization algorithm

association of a timing information (timestamp) to an event



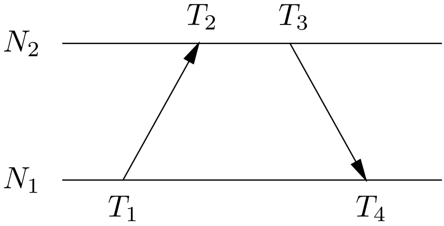
- the Network Time Protocol (NTP) is the most common method to synchronize the clocks of several machines (computers) connected to a network (Internet or LAN)
- it has a layered client-server organization
- it is based on the exchange of UDP message between the client who requests the synchronization information, and the server who provides such information
- it achieves an accuracy of around 10 ms for computers connected to the Internet, and up to 200μs if connected to a LAN



1. client N_1 sends a message containing the timestamp T_1
2. server N_2 receives the message and timestamps it with T_2
3. after the processing, N_2 replies with a message containing all the timestamps, plus T_3, i.e. the timestamp associated to the outgoing reply
4. N_1 receives the message, and timestamps it with T_4

at this point, N_1 knows all the four timestamps T_1, T_2, T_3 and T_4

NTP and clock correction



the following values can be calculated:

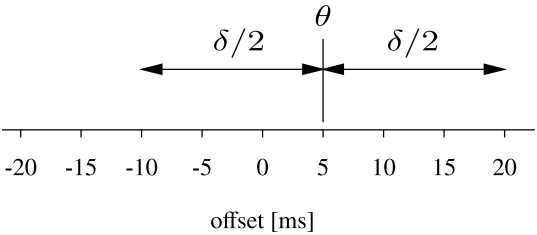
round-trip delay

$$\delta = (T_2 - T_1) - (T_3 - T_4)$$

offset

$$\theta = \frac{(T_2 - T_1) + (T_3 - T_4)}{2}$$

NTP and clock correction

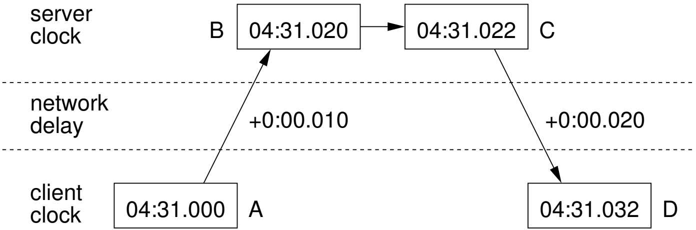


- the clock of the client has offset 0
- it can be proved that for the exact correction θ_0 it holds

$$\theta - \frac{\delta}{2} \leq \theta_0 \leq \theta + \frac{\delta}{2}$$

- the offset is instead the correction that most probably approximates the best correction with the available information

NTP and clock correction: example



- in the example, $\delta = 0.03$s and $\theta = 0.005$s
- it holds that $-0.01 \leq \theta_0 \leq 0.02$, i.e., $0.005 - 0.03/2 \leq \theta_0 \leq 0.005 + 0.03/2$
- without additional information, the NTP applies the correction of 0.005s
- to obtain the exact correction, the correction should have been equal to $\theta_0 = 0.01$s

Correction interval of the NTP

- let x be the true time interval between the departure of the message A and its arrival B (network delay)
- if θ_0 is the real delay of B relative to A, it holds that $x + \theta_0 = T_2 - T_1$
- since $x > 0$, it holds $x = (T_2 - T_1) - \theta_0 \geq 0$, i.e., $\theta_0 \leq (T_2 - T_1)$
- similarly, if y is the true time interval between the departure of the message C and its arrival D, θ_0 is the exact offset of D relative to C, it holds $y - \theta_0 = T_4 - T_3$
- since $y > 0$, it holds $y = \theta_0 + (T_4 - T_3) \geq 0$, i.e., $\theta_0 \geq (T_3 - T_4)$
- it is obtained $(T_3 - T_4) \leq \theta_0 \leq (T_2 - T_1)$

Correction interval of the NTP

now, it is necessary to prove that

$$(T_3 - T_4) \leq \theta_0 \leq (T_2 - T_1) \quad \Leftrightarrow \quad \theta - \frac{\delta}{2} \leq \theta_0 \leq \theta + \frac{\delta}{2}$$

which is done considering the following rewriting of previous equations:

$$T_3 - T_4 = \frac{T_2 - T_1}{2} + \frac{T_3 - T_4}{2} - \left(\frac{T_2 - T_1}{2} - \frac{T_3 - T_4}{2}\right) = \theta - \frac{\delta}{2}$$

$$T_2 - T_1 = \frac{T_2 - T_1}{2} + \frac{T_3 - T_4}{2} + \left(\frac{T_2 - T_1}{2} - \frac{T_3 - T_4}{2}\right) = \theta + \frac{\delta}{2}$$