Robotics
Errors and compensation

Tullio Facchinetti
<tullio.facchinetti@unipv.it>

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http://robot.unipv.it/toolleeo

Problems in sensors measurement

the most prominent problems of sensors arise from

- nonlinearity (in the time and frequency domain)
- noise
- drift of parameters
- cross-sensitivity

Nonlinearity

> the nonlinearity arises from the transfer function that puts into relationship the input value to the measured numerical value

it suffices to eliminate the nonlinearity in the desired sensing range

- the nonlinearity was a worse problem in the era of analog electronics
- nowadays, discrete electronics components inherently solve most of problems related to nonlinearity

Nonlinearity in the frequency (time) domain

usually due to undesired effects of the material used to build the transducer

- dissipation (resistance)
- memory effects (condenser)
- inertial effects (inductance)

the above effects produce high derivative terms in the transfer function (more poles and zeros)

Compensation of nonlinearity

the compensation is theoretically simple: zeros can be used to compensate undesired poles and vice-versa

in practice, the compensation may be tricky due to non-perfect cancellations; it may jeopardize

- stability
- sensitivity

> the construction of a digital filter is simple and automatic procedures are available to do it

- the mobile average is a simple example of low pass filter
- the implementation of a filter requires adequate computing power
- a digital filter requires a model of the sensor, which can be obtained from
  - a theoretical model
  - measurements on the device
- both methods are inherently approximated

Nonlinearity: compensation

techniques based on

- polynomial functions
- Lookup Table (LUT)

Nonlinearity: compensation

in	sample	out
0.0	0.00000	0.0
1.0	0.50000	1.0
2.0	0.70711	2.0
3.0	0.86603	3.0
4.0	1.00000	4.0
5.0	1.11803	5.0
6.0	1.22474	6.0
7.0	1.32287	7.0

let us assume that

- the measurement of the input value `in` produces the numerical value `sample`
- from `sample` it is possible to obtain the corresponding `out` value (the measurement)

> the relationships `in-sample` and `sample-out` are nonlinear

Nonlinearity: compensation

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the compensation of the nonlinearity of the transfer function can be done using a polynomial function

noting that

$$\mathrm{sample} = \frac{1}{2}\sqrt{\mathrm{in}}$$

it holds

$$\mathrm{out} = 4 \cdot \mathrm{sample}^2$$

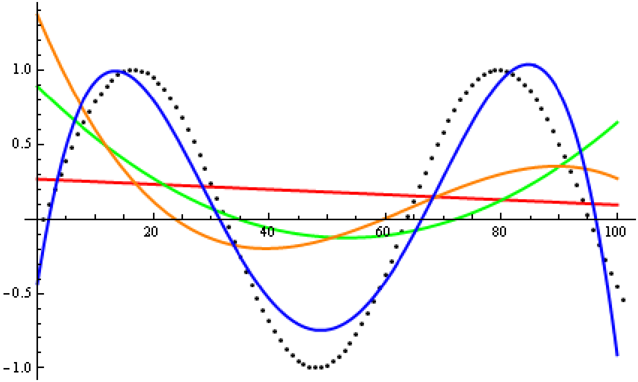
Problems with polynomial functions

however, how can the nonlinearity be compensated if the relationship can not be easily approximated by a polynomial?

in	sample	$4 \cdot \mathrm{sample}^2$
0.0	0.00000	0.00000
1.0	0.45000	0.81000
2.0	0.72711	2.11476
3.0	0.82603	2.72930
4.0	1.10000	4.84000
5.0	1.19803	5.74110
6.0	1.24474	6.19751
7.0	1.42123	8.07957

- using the relationship $\mathrm{out} = 4 \cdot \mathrm{sample}^2$ a wrong measurement is obtained $(\mathrm{out} \neq \mathrm{in})$
- the function may be approximated by a polynomial function

non-linear functions can be approximated by curve fitting



source: Wikipedia

The Look-Up Table

if the approximation using polynomial functions is not accurate enough, the table can be used as a Look-Up Table (LUT) to associate the right value of sample to that of out

- the LUT can be filled with values measured in a calibration phase
- intermediate values in the LUT can be found by interpolation

The Look-Up Table

- the output of the A/D converter is made by M bits
- the LUT is an array of 2^N memory cells, where N is an integer value
- each element of the array stores the expected output value for each digital value provided by the A/D converter

> in previous examples it was $N = 3$
> since the array was made by $2^N = 8$ cells

Calculations using a LUT

the steps required to use a LUT are the followings:

- given a binary data X provided by the A/D converter, a binary mask $\mathrm{mask} =$"$1\ldots10\ldots0$" is applied, i.e., the bit-wise operation $X_{mask} = X \,\mathrm{AND}\, \mathrm{mask}$ is performed
- the mask is composed by N values "1" and $M - N$ values "0"
- X_{mask} is right-shifted of $M - N$ bits, i.e., $X_{shift} = X_{mask} >> (M - N)$
- X_{shift} is used as an index in the LUT

> in practice, the N most significant bits of x are used to access the LUT; in other words, the $N - M$ less significant bits are discarded

LUT: example of usage

the considered system is composed by

- a $\mathrm{LUT}[64]$ array made by 64 cells, i.e., $N = 6$
  (C language notation: valid indexes are in the interval [0..63])
- the A/D converter provides 8 bit values, thus $M = 8$

let us assume the value $X = 10010110$ is read from the A/D

- $\mathrm{mask} = 11111100$
- $X_{mask} = X \,\mathrm{AND}\, \mathrm{mask} = 10010100$
- $X_{shift} = X_{mask} >> 2 = 00100101$
- the value $00100101 = 37$ is used as index in the LUT
- in other words, $\mathrm{LUT}[37]$ will be the value used in the processing

Considerations on the Lookup Table

the dimensioning of a LUT is a trade-off among the following aspects:

- processing speed
- accuracy of the conversion (number of bits of the output value)
- memory footprint of the array

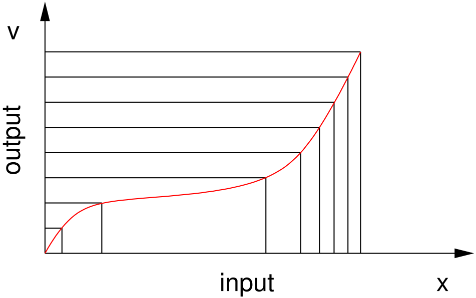
$M = N$

- there is one value in the LUT for each digital output value from the A/D converter
- corresponds to the highest accuracy
- highest memory footprint

$N < M$

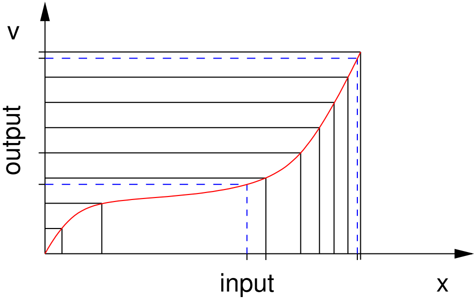
- saves memory space
- less accuracy in the input-output mapping
- in this case, there are two possibilities:
  - the introduced error is neglected $\rightarrow$ no further processing is required $\rightarrow$ more speed
  - adjacent values in the LUT are interpolated $\rightarrow$ the accuracy is improved $\rightarrow$ additional computation and less speed

Dead-band nonlinearity



$v = f(x)$ is the measured voltage as a function of the input value

Dead-band nonlinearity

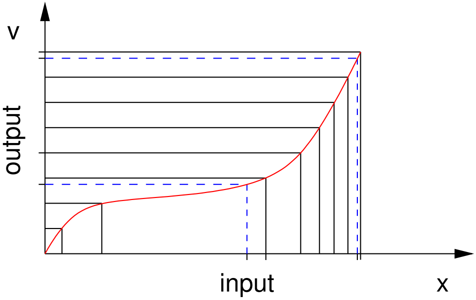


since

$$e_x = \frac{e_v}{f'(x)}$$

a small error e_v on the measured value can have a great impact on the accuracy of the measure

Dead-band nonlinearity



- this type of nonlinearity is "pathological", since it depends on the shape of the tranfer function
- there are no good compensation techniques
- even using (computationally) heavy algorithms, large accuracy losses can not be avoided

the noise
is made by every undesired signal

- the noise is associated, in general, with random noise (i.e., white noise)
- the noise affects every possible measurement, since it is related with the intrinsic thermal agitation of atoms

$1/f$ noise (pink noise)

> the amplitude of this type of noise is inversely proportional to the frequency of the signal

1. its source is still debated
2. in some cases, it is due to variations in atomic layers of the external boundary of sensor bodies
3. sampling issues are exacerbated when measuring signals having frequency close to 0
4. it is immune to filtering based on averaging, since those latter are essentially low-pass filters

Drift of parameters

it is due to small changes in the atomic structure of the material used to build the sensor

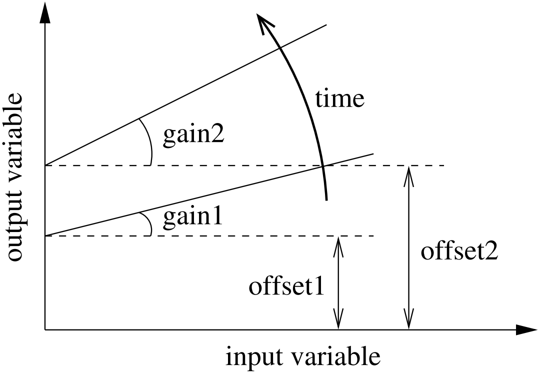
- the crystal structure of the material may change if mechanical stress is applied
- the atoms of two adjacent materials can diffuse and change the properties of one component
- the external surface of a transducer is subject to the chemical-physical influence of gases and other fluids (e.g., oxidation)

Drift of parameters

> in some applications the sensor can not be considered a time-invariant system as could be modeled in a first approximation

- the drift can affect the offset and/or the gain of a transfer function
- to compensate the drift of the gain the only possiblity is to perform periodic calibrations, better if they can be done automatically

Drift of parameters



example of drift that affect both offset and gain

Cross sensitivity

sensors are always sensitive
to more than one input variable

- in practice, all sensors are affected by the temperature (at least)
- temperature variations can produce non-linear variations of both offset and gain in the transfer function

available methods can be classified as follows:

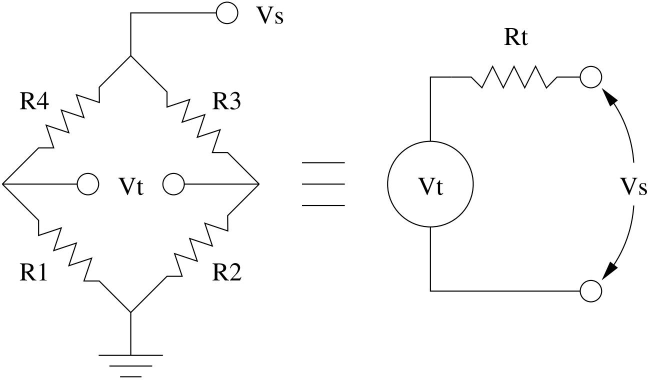
- structural compensation
- calibration
- compensation through monitoring
- deductive compensation

this classification is proposed in

J.E. Brignell, "Sens. Actuators", **10** 249–261 it is based on an adeguate (structural) design and engineering of sensor components

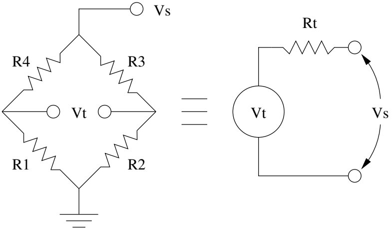
- one fundamental guideline to achieve structural compensation is to enforce the symmetry of components
- the idea is that the target variable must produce a differential signal, while undesired variables shall generate a common mode signal

e.g.: the design of the unbalanced double T transducer in the MEMS accelerometer compensates the pitch accelerations



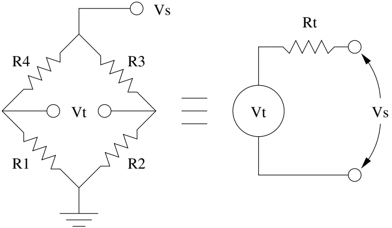
the Wheatstone bridge design leverages structural compensation principle based on the symmetry of its components

The Wheatstone bridge: characteristics



- it is one of the most common circuits for signal measurement
- it improves the quality of the measurement by achieving higher sensitivity
- allows to perform a differential measurement
- reduces the impact of the common mode signal noise

The Wheatstone bridge: characteristics



when the circuit is not balanced:

$$V_t = V_s\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right)$$

$$R_t = \frac{R_2 R_3}{R_2 + R_3} + \frac{R_1 R_4}{R_1 + R_4}$$

in balanced conditions:

$$\frac{R_4}{R_1} = \frac{R_3}{R_2}$$

if all resistors have the same value, then V_t is null let's suppose that

- the sensor includes a Wheatstone bridge having 4 resistors with nominal resistance R
- 3 resistors are passive elements
- the R_1 resistor changes its value of ΔR due to the variation of the measured variable (e.g., strain gauge, potentiometers, etc.)

the output voltage becomes

$$V_t = \left(\frac{\Delta R}{4R + 2\Delta R} \right) V_s$$

where V_s is the known supply voltage

note that the equation

$$V_t = \left(\frac{\Delta R}{4R + 2\Delta R}\right) V_s$$

is non-linear in ΔR...

but if $\Delta R << R$ then a linear equation is obtained as a result:

$$V_t = \frac{\Delta R}{4R} V_s$$

Wheatstone bridge: structural compensation

- the bridge is used to compensate the effect of the temperature on the measurement
- it assumes that the temperature affects all the resistors in the same manner
- there is the same variation of resistance due to the temperature by all resistors

even in case of changing environmental temperature, the output of the bridge remains balanced, and only the input variable affect the variation of the output voltage

Compensation through calibration

usually the structural compensation is not enough

- a sensors always presents some defects due to the manufacturing process
- different items of the same devices are always slightly different

each device should be individually calibrated

scenario:

- a manufacturing line builds devices that should be perfect copies
- the devices differ due to manufacturing inaccuracies
- the optimal solution would be the calibration of each single device

> the compensation through calibration can be an expensive solution

Compensation through calibration

- to overcome the problem some components are inserted in the device that allow an easy calibration after the manufacturing (i.e., it is chea to be done)

- in case of intelligent sensors, the device is equipped with programmable memoris (e.g., EPROM) that are loaded with the calibration parameters (e.g., the values of a LUT)

Compensation thought monitoring

this method is specific to intelligent sensors

- the sensor include some components to monitor (measure) the interesting variables – usually the temperature – in order to compensate their effects
- once the disturbance has been measured, its effect is compensated using a model derived through calibration performend in the factory

e.g.: the “temperature compensated” oscillators measure the temperature of the crystal to adequately compensate its variation

Deductive compensation

> deductive compensation is mandatory
> when the system is not physically reachable

examples of such systems are:

1. the explosion room of a motor
2. a nuclear reactor
3. the human body

- in this case the compensation is necessarily based on a model of the system
- since all models are somehow approximated (especially in case of complex systems), this method is used as a last solution when other methods do not apply