Robotics
Intelligent sensors (part 2)

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http://robot.unipv.it/toolleeo

Pressure measurement

static pressure is a force applied to an area

$$P = F/A$$

- its measurement is usually associated with fluids (liquid or gases)
- it is related to the height of a liquid in a vessel

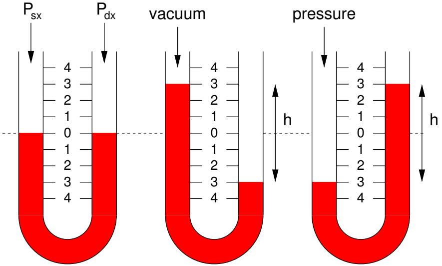
definitions:

- **absolute pressure**: referred to the vacuum pressure
- **relative pressure**: referred to some known reference pressure, often the atmospheric pressure (gauge pressure)
- **differential pressure**: measures the difference between two pressure values P_1 and P_2

the following relationship holds:

$$\text{absolute pressure} = \text{relative pressure} + \text{atmospheric pressure}$$

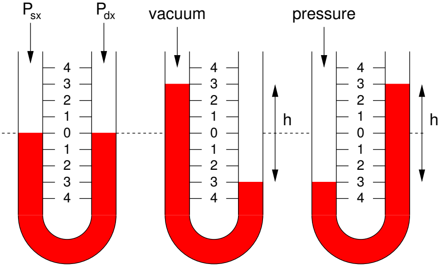
Pressure sensor: U-tube manometer



$$P_{dx} - P_{sx} = h\rho g$$

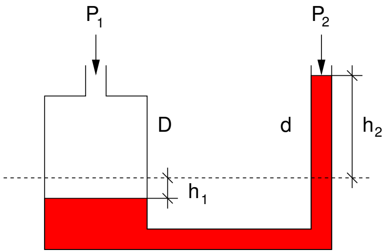
- P_{dx} and P_{sx} are pressure applied to the two sides of the tube
- ρ is the fluid density
- g is the gravity acceleration

Pressure sensor: U-tube manometer



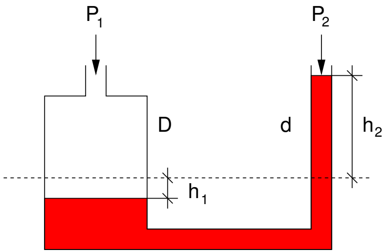
measured pressure	reference value
absolute	vacuum
relative	atmospherical
differential	reference

Pressure sensor: U-tube manometer



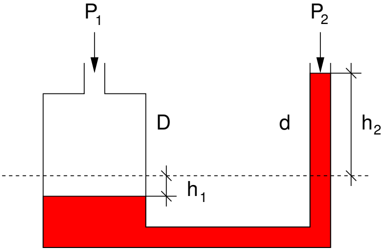
- two connected cylindrical tubes (diameters D and d)
- the measured value is h_2
- the amount of liquid $V_1 = V_2$ that transits from the left to the right tube is

$$V_1 = h_1 \frac{\pi D^2}{4} \qquad V_2 = h_2 \frac{\pi d^2}{4}$$



i.e., the height h_1 is

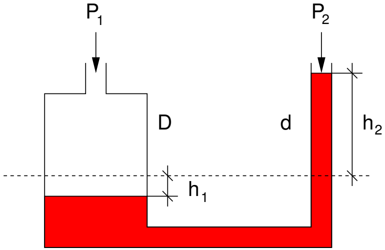
$$h_1 = \frac{V_2}{\pi D^2/4} = \frac{h_2 \pi d^2/4}{\pi D^2/4} = h_2 \left(\frac{d}{D}\right)^2$$



since the difference between two pressures is proportional to the height of the liquid, it holds

$$p_1 - p_2 = \rho g \left(h_2 + h_2 \left(\frac{d}{D} \right)^2 \right) = \rho g h_2 \left(1 + \left(\frac{d}{D} \right)^2 \right)$$

Pressure sensor: U-tube manometer



- can measure pressures from 1 to 7000 bar
- the size (section) of tubes is related to the range of pressure to measure
- this affects:
  - the maximum height of the liquid
  - the size of the device

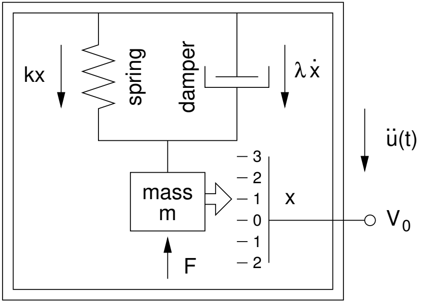
- the U-tube manometer does not directly provide the value of the measured pressure as an electric signal, to be used in a control loop
- the electric signal can be obtained by measuring the height of the liquid using a linear displacement sensor and a float
- a common approach is to use an ultrasound proximity sensor to measure the height of the liquid in the tubes

the acceleration is the second derivative of the position

$$V = S/t \quad \rightarrow \quad a = V/t \quad \rightarrow \quad a = S/t^2$$

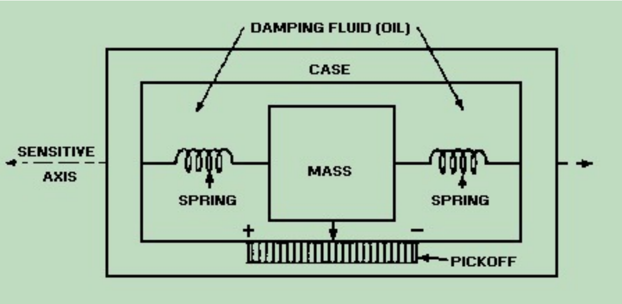
$$V = \frac{dS}{dt} \quad a = \frac{d^2 S}{dt^2}$$

- this observation suggests the possibility to calculate the acceleration by measuring a displacement and to compute the second derivative w.r.t the time
- a distance could be measured using an odometer or displacement sensor
- this method **is never adopted** due to the high frequency noise amplification produced by the derivative
- all existing accelerometers are based on mass-spring-damper systems



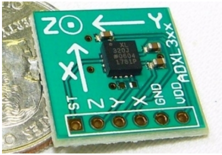
- m is the value of the mass $[kg]$
- λ is the damping coefficient $[Ns/m]$
- k is the spring constant $[N/m]$
- $\ddot{u}(t)$ is the acceleration of the chassis, which can be different from $\ddot{x}$ $[m/s^2]$
- x is the displacement of the mass w.r.t. the chassis $[m]$
- V_0 is the measured output voltage $[V]$

example of practical implementation of an accelerometer:



- the case is filled with oil, which provides the damping effect
- the mass can move on one single direction
- the sensing is done using a capacitive linear position transducer

comparison between two technologies:



the differential equation that put into relationship the aforementioned values is

$$F = m\ddot{x} + \lambda\dot{x} + kx = ma$$

the natural (non-damped) frequency of the system is defined as

$$\omega_0 = \sqrt{k/m}$$

the damping factor is defined as

$$\zeta = \lambda/2\sqrt{km}$$

the equation can be rewritten as

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega^2 x = \ddot{y}(t)$$

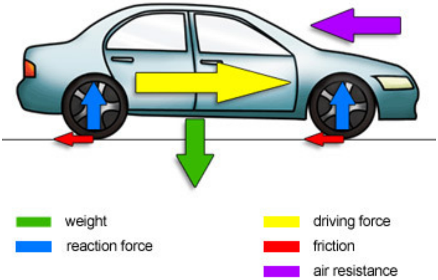
with an harmonic input – such as $u(t) = u_0 \sin(\omega t)$ – the transfer function can be rewritten by replacing the derivatives with s:

$$\frac{X}{\ddot{U}} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \left(2\zeta\frac{s}{\omega_0}\right) + 1}$$

by assuming that the sensor converts the relative position into a voltage such as $v_0 = S_x \cdot x$, the result is

$$\frac{V_0}{\ddot{U}} = \frac{S_x}{\left(\frac{s}{\omega_0}\right)^2 + \left(2\zeta\frac{s}{\omega_0}\right) + 1}$$

where S_x is the sensitivity of the sensor expressed in [volt/div] different kind of forces to measure



source: `www.bbc.co.uk`

source: Star Wars
(harder to measure!)

there are two different sources of a measurable force:

1. the inertia can be leveraged when an object is free to move
2. force counter-balancing can be used to measure a force applied to a constrained body

How to measure a force

the measurement of a force
is always done indirectly

different methods:

1. balancing the unknown force with the gravity (e.g., two-pan balance)
2. measuring the acceleration of a body having known mass
3. balancing the unknown force with a magnetic force and measuring the required power/energy
4. measuring the pressure generated by the force on a known area
5. measuring the deformation of an elastic body (e.g., spring balance)

> the more recent and reliable techniques are based on elastic properties of adequate materials

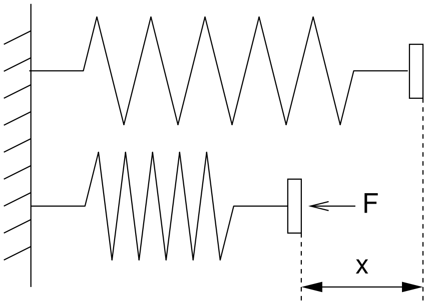
the physical Law that is behind this methods is

$$F = kx$$

where

- F is the unknown force to be measured
- x is the displacement to measure
- k is the elastic constant of the transducer

a simple example is provided by a spring system

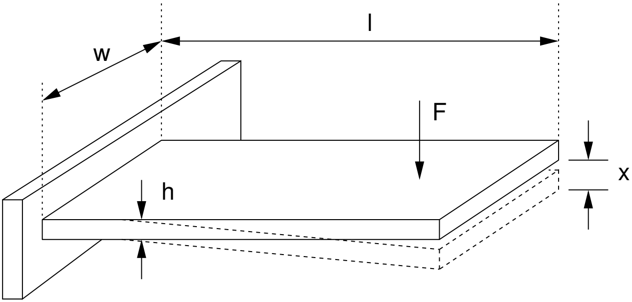


$$F = kx$$

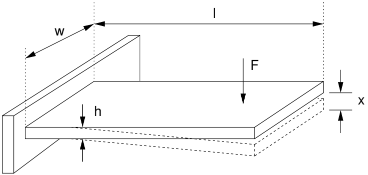
the cantilever is a beam anchored at only one end

cantilevers are very appreciated by architects and designers

sources: www.dezeen.com / www.bonluxat.com the cantilever is a beam anchored at only one end



the load is kept by the momentum and the resistance to the cut in the anchor point

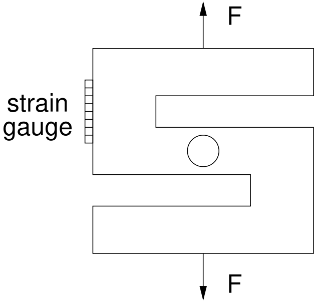


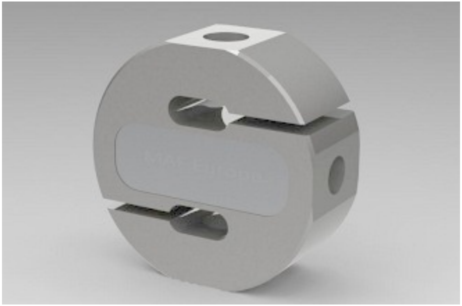
if small deflections are assumed for the beam, the following equation puts into relationship the parameters of the cantilever:

$$x = \frac{4Fl^3}{Ewh^3}$$

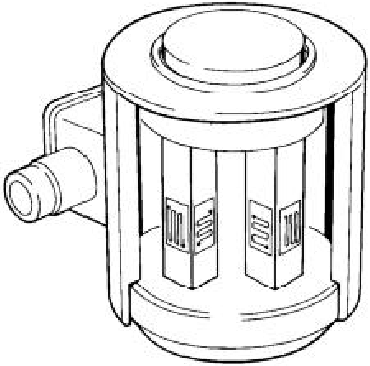
- F is the force to measure
- l is the length of the beam
- w is the width of the beam
- h is the thickness of the beam
- E is the Young's modulus of the material

The load cell





Load cell: example

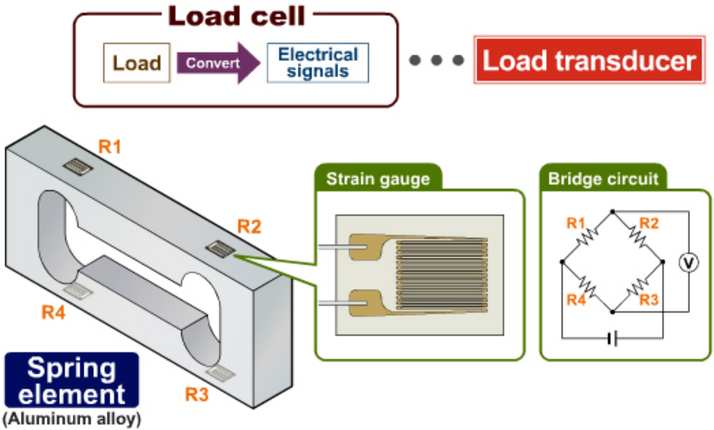


notice the strain gauges applied to the core element

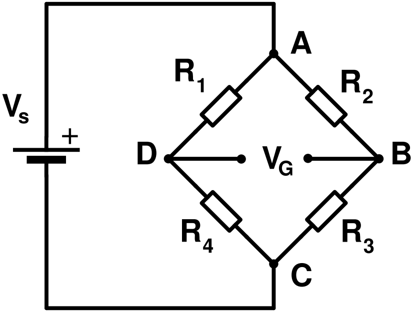
Load cell

load cells are available in many shapes and sensing ranges

the sensing is actually made by a Wheatstone bridge where strain gauges are inserted

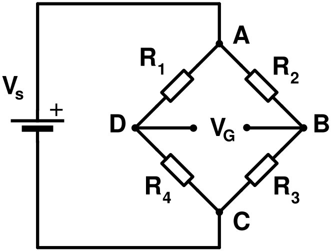


source: http://www.ishida.com/



$$V_G = V_s \left(\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} \right)$$

V_G is null if the resistances are perfectly balanced
i.e., $R_1 = R_2 = R_3 = R_4$

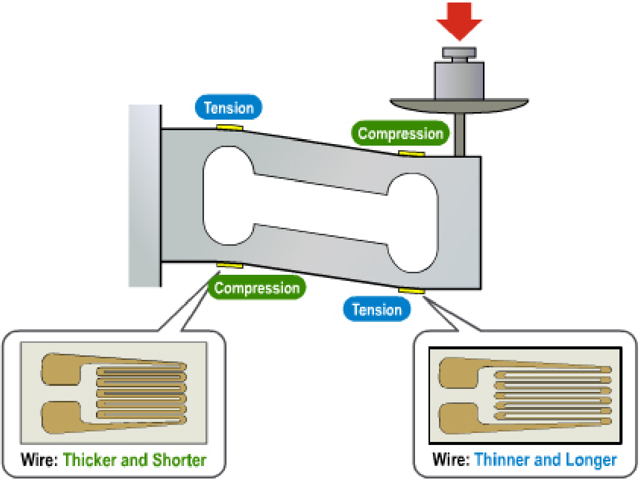


$$V_G = V_s \left(\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} \right)$$

The measurement is done by assuming that one or more resistors change their resistance depending on the variation of the physical quantity to measure.

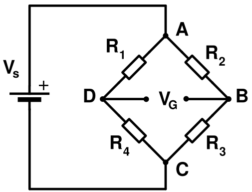
The scheme is useful to compensate the effect of the temperature on the measurement: when the temperature changes, all the resistances change by the same amount, and the output voltage remains unaffected by the change.

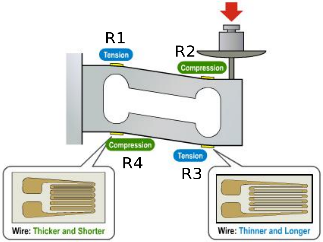
Load cell



source: http://www.ishida.com/

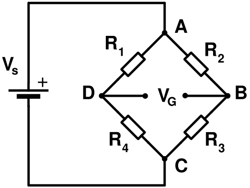
Load cell and Wheatstone bridge

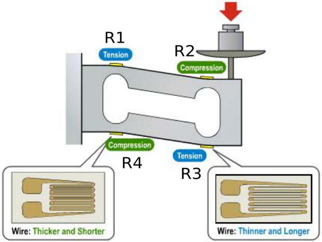




$$V_G = V_s \left(\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} \right)$$

$R2$ and $R4$ decrease (by Δ_R)
$R1$ and $R3$ increase (by Δ_R)





$$V_G = V_s \left(\frac{R_3 + \Delta_R}{(R_2 - \Delta_R) + (R_3 + \Delta_R)} - \frac{R_4 - \Delta_R}{(R_1 - \Delta_R) + (R_4 + \Delta_R)} \right)$$

- $R2$ and $R4$ decrease by Δ_R
- $R1$ and $R3$ increase by Δ_R

- $R_3/(R_2 + R_3)$ increases
- $R_4/(R_1 + R_4)$ decreases

denominators remain unchanged

Let's assume $R_1 = R_2 = R_3 = R_4 = R$

$$V_G = V_s \left(\frac{R + \Delta_R}{(R - \Delta_R) + (R + \Delta_R)} - \frac{R - \Delta_R}{(R - \Delta_R) + (R + \Delta_R)} \right)$$

$$V_G = V_s \left(\frac{\Delta_R}{R + R} - \frac{-\Delta_R}{R + R} \right)$$

$$V_G = V_s \frac{2\Delta_R}{2R} = V_s \frac{\Delta_R}{R}$$

The output voltage is proportional to the variation of resistance, which is related to the amount of deformation and thus to the force to measure.