Modeling and real-time control of an industrial air multi-compressor system

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***Abstract*—This paper presents a control algorithm for an air multi-compressor system. The goal is to achieve adequate performance in terms of air pressure regulation by properly coordinating a set of compressors driven by fixed speed motors. The coordination is required to impose an upper bound to the activation frequency of electric drives. A multi-compressor system is intended to be a viable alternative to compressor systems based on Variable Speed Drives (VSD) operated by inverters, which suffer of several technical and economic drawbacks. The control strategy is based on the evaluation of the timing associated to activations/deactivations of each compressor. Such evaluation is determined by the values of physical variables that determine the system behavior, including air flows, pressures and temperature. The periodic measurement of the actual pressure is performed to dynamically adjust the estimation of relevant time instants in case of variations of working conditions. The algorithm takes into account the dynamics of the air pressure, as well as timing constraints on the minimum period between two subsequent activations of each compressor. The effectiveness of the multi-compressor solution is evaluated by simulation.**

I. INTRODUCTION

Compressed Air Systems (CASs) are widespread in nowadays industries due to their flexibility, since they allow to implement many manufacturing processes using a safer energy source w.r.t. electricity. For this reason, pneumatic actuators are often better solutions than electric drives in machinery and industrial equipment in general. With the growing attention to energy-aware technologies in industrial applications, CAS are receiving increasing interest. The operational cost of a CAS can be up to 70% to 90% of the total electricity bill for specific users [1], while an average of 30-40% is common for typical users. The energy cost is due to many factors. The foremost factor is the energy loss due to the low efficiency of air compression, usually below 10-15% [2].

A CAS is usually equipped with multiple compressors. Their combined capacity is tailored to meet the maximum plant air demand, as well as to ensure the supply reliability, i.e., to maintain the internal pressure of the pneumatic circuit within a predefined working range. In recent years, Variable Speed Drives (VSDs) have been increasingly adopted as a technological solution to achieve a fine-tuned pressure control and to address the energy issue in CASs. Basically, a VSD is an electric motor equipped with some electric/electronic components (usually an inverter) that allow to regulate the motor speed. Such speed is reduced when the load requires less power or torque. The speed adaptation allows to reduce the power demand when possible. In a CAS, a VSD is often proposed as a solution to provide a variable and adjustable air flow to cope with the pressure fine-tuning, i.e., to accurately match the amount of demanded outlet air flow with the produced inlet flow.

On the other hand, the use of VSDs poses several issues, both technical and economical. The most prominent issue is the cost of the inverter required to implement a VSD. A related problem is the relatively short lifetime of an inverter, which may need to be replaced around every 5 years. The replacement is expensive, and it is often made difficult by the rapid sell out of older models in stock, which are replaced by newer models that may be not retro-compatible. A "strategic" commercial issue, which becomes very relevant in practical distribution and installation of CASs based on VSDs, is related to the scarce availability of the technology, especially in less developed geographic areas and countries. Moreover, vendor lock-in issues may arise due to the limited amount of manufacturer of VSDs. The use of fixed speed drives to implement a multi-compressor system addresses the above issues, since these drives are cheaper, more widely spread and robust than VSDs. Technically speaking, the electronics of VSDs is known to generate spurious signals on the supply line, which may disturb the operation of other devices and potentially damaging other power electronics components. An additional advantage of a multi-compressor vs a VSD-based solution is related to fault tolerance: when the motor of a VSD stops working properly, the entire CAS driven by the VSD is compromised (unless redundant backup systems are installed); instead, when one motor of a multi-compressor breaks, the remaining compressors may continue their operation at reduced service.

The above technical and economic issues have been identified by evaluating the current state-of-the-art of the air compressor market. They motivate the investigation of a solution for industrial air compression that does not rely upon the integration of a VSD in the system. The goal of this paper is to describe a CAS based on a tightly coupled set of fixed speed compressors (referred as multi-compressor in the reminder of this paper) that aim to provide pressure fine-tuning comparable to VSD-based CASs, while limiting the loss of energy saving.

Standard fixed speed compressors are characterized by 3 working modes, referred as *off*, *load* and *unload*. The characteristics of different modes are the following:

- in off mode, the compressor is completely switched off; neither electricity is consumed nor output air flow is generated;
- in load mode the power demand is 100%, and the compressor generates a constant amount of output air flow;
- in unload mode the power demand is typically around 85% less than the power consumed in load mode, while the air flow is null; in practice, the electric motor of the compressor is decoupled from the pump: it runs unloaded, thus pumping no air.

A typical coordination strategy for controlling a set of compressors is based on the periodic switch off of some selected machines (on-off control). This approach achieves the best possible performance in terms of energy efficiency, since compressors are kept in off mode as long as possible. However, there are technical limitations to the frequency of transitions from the off to the load mode. In fact, during such transitions, the motor absorbs an electric current that can be up to 3 times higher than the current absorbed while running at full speed[1]. This startup current may overheat the insulations and other sensible parts, leading to a quick wearing of components, which in turn reduces the device's lifetime and/or increase the frequency of maintenance. For above reasons, it is recommended that the maximum frequency of off-to-load transitions is limited to 12 switches/hour. In practice, this approach can only be used in a proactive scenario where some reliable forecasts on future compressed air usage are available.

A similar approach leverages load-unload transitions in order to regulate the pressure. Despite the power consumed in unload mode is not as low as in off mode, the overall required energy is reasonably low. Moreover, electro-mechanical coupling technologies are available that allow relatively frequent unload-to-load transitions. For example, a switch every 10 seconds can be tolerated.

This paper proposes a coordination strategy for a multi-compressor unit based on this latter approach. The goal of the control policy is to maintain the air pressure of the pneumatic circuit within the pre-defined desired working range, while maximizing the period between two consecutive unload-to-load switches of each compressor. A dedicated control algorithm is proposed to determine the configuration of active compressors at any given time, and to schedule relevant system events such as pressure measurements, control actions, and the activation of the desired set of active compressors. Simulated results show that the proposed control policy guarantees the required pressure and achieves sufficiently low unload-to-load transitions under realistic working conditions.

The paper is organized as follows. Section II provides an overview of related works. The model of the physical system is presented in Section III, together with the objective of the control. A preliminary elaboration of physical equations related to the control algorithm is done in Section IV, while Section V describes in details the Finite State Machine that encapsulates the control logic. Section VI shows the simulation results that assess the performance and the behavior of the control policy. Section VII concludes the paper.

[1]This is an optimistic value, which is obtained by using special soft start electronic devices to regulate the startup.

II. Related works

Despite the relevance of CASs in the industrial domain, there are relatively few scientific papers published on the subject. In fact, most of information are jealously conserved by companies operating in the market of compressed air. However, there are some works concerning the evaluation and the control of CASs, especially dedicated to the more recent technologies based on VSDs.

The authors of [3] discuss benefits and issues, both economical and technological, of VSD-based CASs. They present the available technologies behind VSDs, and investigate their application by means of real case studies. A comparison between VSD-based and load-unload systems is provided in [4]. In [5] a control method based on neural networks is proposed, while in [6] the proposed control approach leverages a Model Predictive Control scheme (MPC). The research in [7] takes into account the instability generated during the operation of the two main types of air compressors: axial and radial machines. A load-unload control scheme is described in [8]. This work is based on the forecasting of the compressed air demand, which is inferred on the basis of a previous monitoring campaign. These information are not leveraged in our proposed method. The frequency conversion technology is adopted in [9] to control a VSD. In [10], the non-linear behavior of a CAS is explicitly modeled and a specific control technique is applied to deal it. The performance of the controller are proved to outperform those of a traditional and commonly adopted PID control scheme. Finally, in [11] an original approach is proposed to save energy by designing a CAS made by one bigger compressor (whose efficiency is higher) instead of several smaller ones, where each compressor is dedicated to operate in a specific air pressure range. Open-close operations of pneumatic valves is scheduled to generate the correct pressure ranges.

All of above papers address the control of single compressors. None of them considers the coordination of more than one compressor to achieve same goal.

III. System model

This section describes the model adopted for the physical system and the corresponding timing parameters used to manage the schedule of compressors activations.

A. The physical system

The physical system is composed by a set of N air compressors $\Gamma = \{c_1, \ldots, c_N\}$ that feed a compressed air circuit having constant volume. The volume of the circuit, denoted with V, includes the volume of distribution lines and the volume of all possible air tanks installed in the system. Compressors are connected in parallel to the same air circuit. They can be independently controlled to regulate their contribution to the total amount of generated compressed air (see Figure 1). Each compressor c_i generates a nominal air mass flow Q_i. The output pressure is compatible with the

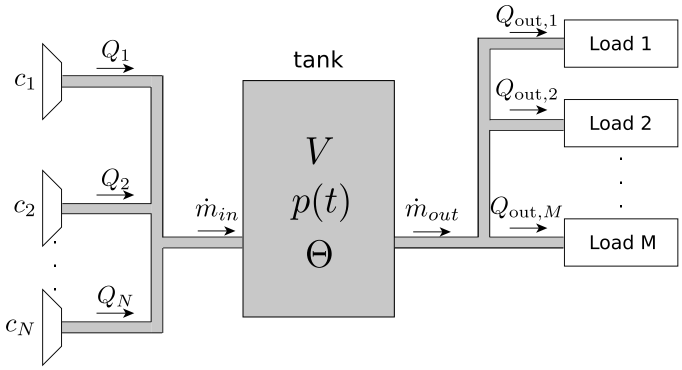


Fig. 1: The physical system including the compressors, the air circuit composed by tanks and pipes, and the loads.

desired system pressure $\overline{p}$, i.e., the set-point pressure. This means that the output pressure of each compressor must be greater or equal to the required pressure set-point. In other words, a compressor whose nominal output pressure is less than $\overline{p}$ would not be able to pump air in the circuit.

The physical quantities of interest related to the compressed air circuit are ruled by the well-known gas state equation:

$$Vp(t) = R^{\text{air}}\Theta m(t) \tag{1}$$

where $R^{\text{air}} = 287\,\text{J/Kg}\cdot\text{K}$ is the air gas constant, $p(t)$ is the actual pressure within the air circuit at time t, $m(t)$ is the air mass contained in the circuit at the same time t, and Θ is the temperature of the air in the circuit, considered constant.

To model the dynamics of the involved physical quantities, the derivative of Equation (1) is considered:

$$\dot{p}(t) = \frac{1}{K}[\dot{m}_{\text{in}}(t) - \dot{m}_{\text{out}}(t)] \tag{2}$$

where $K = \dfrac{V}{R^{\text{air}}\Theta}$.

Equation (2) is known as *mass conservation law*. The equation puts into relationship the pressure variation with the variation of air mass within the circuit due to the input and output air mass flows, denoted with $\dot{m}_{\text{in}}(t)$ and $\dot{m}_{\text{out}}(t)$ respectively.

Let be $s_i(t) : R^+ \rightarrow [0, 1]$ the *activation function*, or *schedule*, associated to the c_i compressor. This function is defined as:

$$s_i(t) = \begin{cases} 0 & c_i \text{ compressor is in unload state at time } t \\ 1 & c_i \text{ compressor is in load state at time } t \end{cases}$$

The activation function $s_i(t)$ is generated by the control algorithm, and determines the load-unload pattern of compressor c_i. The set of loaded compressors at time t is denoted as $\Gamma^1(t) = \{c_i | s_i(t) = 1\}$. Similarly, the set of unloaded compressors is $\Gamma^0(t) = \{c_i | s_i(t) = 0\}$. It holds $\Gamma^0(t) \cup \Gamma^1(t) = \Gamma(t)\ \forall t$. The actual configuration of the multi-compressor, composed by loaded and unloaded compressors at time t, will be denoted by $\Gamma^{01}(t)$.

Given the definition of $s_i(t)$, the input air mass flow $\dot{m}_{\text{in}}(t)$ can be expressed as the sum of the air mass flows generated by all the active compressors at time t. More formally:

$$\dot{m}_{\text{in}}(t) \equiv q_{\text{in}}(t) \equiv \sum_{i=1}^{N} Q_i s_i(t) \tag{3}$$

The maximum air flow simultaneously requested by all the loads in the system is denoted as Q_{out}. Formally:

$$Q_{\text{out}} = \sum_{j=1}^{M} Q_{\text{out,j}} \tag{4}$$

Q_{out} is a known quantity, since the characteristics of all loads attached to the air circuit are assumed to be available. Given Equation 4, the system output flow is defined as

$$\dot{m}_{\text{out}}(t) \equiv \mu(t) Q_{\text{out}} \tag{5}$$

where $\mu(t) \in \mathbb{R}$, with $0 \leq \mu(t) \leq 1\ \forall t$, is the aggregated utilization factor of the system. The utilization factor represents the fraction of air flow that is requested at time t by the whole system. It is determined by the specific loads that are active at time t. We assume that $\mu(t)$ can not be controlled; it only depends from the use of compressed air made by the plant.

To ensure the possibility to control the system internal pressure in worst case conditions, the following constraint must hold:

$$Q_{\text{in}} = \sum_{i=1}^{N} Q_i \geq Q_{\text{out}} \tag{6}$$

The above condition refers to the worst case situation where a constant outlet air flow equal to Q_{out} is continuously requested. In this case, the total amount of inlet flow Q_{in} must be at least equal to the requested outlet flow to allow the compressors to maintain a constant pressure in the system. For instance, if $Q_{\text{in}} = Q_{\text{out}}$, when the outlet flow continuously requests the maximum amount of air, then all compressors must be continuously active.

We denote with τ^{delay} the minimum time frame between two consecutive unload-to-load transitions of one compressor. An upper bound to τ^{delay} is imposed due to electrical/mechanical characteristics of the electric drive. The terms $t_i^{\text{last\_on}}(t)$ and $t_i^{\text{next\_on}}(t)$ denote the absolute time instants when, respectively, the latest unload-to-load transition occurred and the next transition is scheduled by the controller for compressor c_i w.r.t. time t. Figure 2 graphically depicts the above quantities.

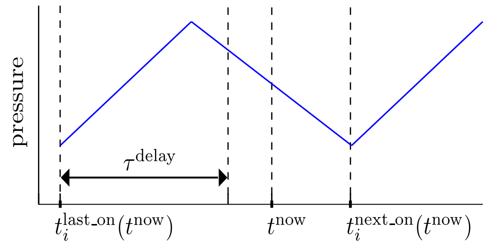


Fig. 2: The meaning of τ^{delay} and related quantities. The values of $t^{\text{last\_on}}$ and $t^{\text{next\_on}}$ refer to the time instant t^{now}.

B. *Objective of the control*

The goal of the control policy is to coordinate the activation of available compressors to maintain the system pressure $p(t)$ within a predefined working range at any time t, while achieving the constraint on τ^{delay}, i.e., the condition that prevent too frequent unload-to-load transitions. Usually, in the industrial domain, the working range of an air compressed system is specified by the set-point $\overline{p}$ and an absolute pressure tolerance $\epsilon_p > 0$. In formal terms, the following condition must be achieved by the controller:

$$\overline{p} - \epsilon_p \leq p(t) \leq \overline{p} + \epsilon_p \quad \forall t > 0 \tag{7}$$

while achieving the condition

$$t_i^{\text{next\_on}}(t) \geq t_i^{\text{last\_on}}(t) + \tau^{\text{delay}} \tag{8}$$

To simplify the specification of the control algorithm, we adopt a more compact notation by defining $p_{min} \equiv \overline{p} - \epsilon_p$ and $p_{max} \equiv \overline{p} + \epsilon_p$. Using those terms, Equation (7) can be rewritten as

$$p_{min} \leq p(t) \leq p_{max}$$

It is worth to note that the constraint expressed by (8) may prevent the possibility to always keep the pressure within the working range. In fact, unfavourable combinations of air requests may require a compressor to switch on too early w.r.t. to the imposed delay. Since the violation of the constraint imposed by τ^{delay} may damage the electric drives, sporadic violations of the air pressure working range are tolerated to guarantee Equation 8.

IV. CONTROL ALGORITHM

To achieve the goal stated in Section III-B, a control algorithm is proposed to coordinate the operation of available compressors. The control logic is fully defined by the Finite State Machine (FSM) represented in Figure 3. The reminder of this section explains in details the meaning and the behavior of the states composing the FSM. To facilitate the comprehension, we first provide a general overview of the proposed control policy.

The control action is based on a continuous sequence of pressure measurements. When the pressure is measured, the coordination algorithm determines whether a control action is required or not. If not required, a new sampling event is scheduled in the future. The time instant of the new sampling event depends on the current system status, in terms of supplied and demanded air flow. A minimum inter-arrival time τ_{min} between two consecutive air pressure measurements is imposed by design. In this way, the measurement task could be implemented as a sporadic task in a real-time operating system [12]. By considering the minimum inter-arrival time, a narrower range $[p_{\text{min\_th}}, p_{\text{max\_th}}] \subset [p_{\text{min}}, p_{\text{max}}]$ can be determined where the normal operations of the multi-compressor are constrained. In practice, the narrowed working range shall not be too close to the working range to satisfy the constraint on the minimum inter-arrival time.

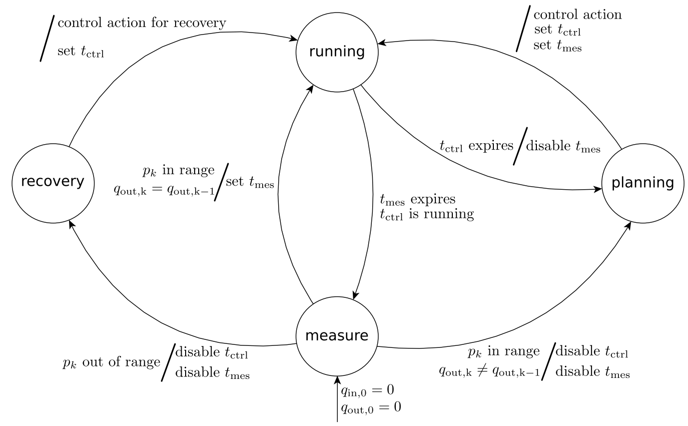


Fig. 3: The FSM that describes the control logic.

On the other hand, if a control action is needed, the coordination strategy selects the right combination of machines to activate, taking into account the constraint imposed by (8) on τ^{delay}. Finally, a recovery policy deals with possible violations of pressure constraints. The same policy is applied to start the system, when the internal pressure is null, in order to enter the normal operation range.

Before providing the detailed explanation of the FSM, some calculations related to the timing dynamics of the physical system are introduced, whose values are used in the control algorithm.

A. *Evaluation of timing parameters*

a) Time to reach a specified pressure: In several steps of the control algorithm, there is the need to calculate the time required to reach a given air pressure starting from known initial conditions.

Assume that at time t_1 the system pressure is p_1. The goal is to reach a final pressure p_2. The unknown value is the time t_2 at which the final pressure will be reached. Let's denote the difference between the inlet and outlet air flows as $M(t) = \dot{m}_{\text{in}}(t) - \dot{m}_{\text{out}}(t)$. Assuming that $M(t)$ has a constant value M during the time interval $[t_1, t_2]$, Equation (2) becomes

$$\dot{p}(t) = \frac{M}{K} \quad \forall t \in [t_1, t_2] \tag{9}$$

By integrating 9 in the interval $[t_1, t_2]$ with respect to time, the following relation is derived:

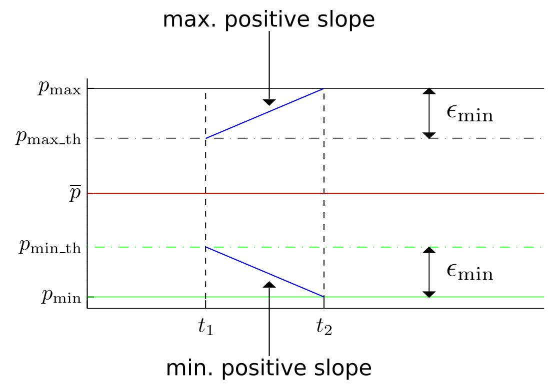


Fig. 4: Geometrical relationships between the quantities managed in the calculations of the inner thresholds.

$$p_2 - p_1 = \frac{M}{K}(t_2 - t_1)$$

The desired time interval $t_2 - t_1$ is obtained as follows:

$$t_2 - t_1 = \frac{K}{M}(p_2 - p_1) \tag{10}$$

b) Calculation of inner thresholds: The control algorithm is based on a sporadic sampling of the air pressure. The minimum time between two pressure measurements will be denoted by the constant value $\tau_{\min}$, which is decided by the system designer. The size of the narrower working range can be determined by imposing $t_2 - t_1 = \tau_{\min}$ and $M = Q_{\text{in}}$ in Equation (10), thus obtaining:

$$\tau_{\min} = \frac{K}{Q_{\text{in}}}\epsilon_{\min} \tag{11}$$

where $\epsilon_{\min}$ is the difference between the limits of the regular working range and the inner one. Figure 4 shows the geometrical relationships between the quantities managed in the calculations.

The maximum slope of the pressure is used in Equation 11, since the guarantee to remain within the working range $[p_{\min}, p_{\max}]$ must be achieved in worst case conditions, i.e., in case of the maximum possible pressure variation. Notice that Q_{in} corresponds to the maximum pressure variation both in case of positive and negative slope. In fact, in case of positive slope, it holds $\dot{m}_{\text{in}} = Q_{\text{in}}$ (all compressors are active) and $\dot{m}_{\text{out}} = 0$ (no air is demanded). On the other hand, in case of negative slope, it holds $\dot{m}_{\text{in}} = 0$ (all compressors are switched off) and $\dot{m}_{\text{out}} = Q_{\text{out}} = Q_{\text{in}}$, corresponding to the maximum air demand (see the considerations after Equation 6). The value of $\epsilon_{\min}$ can be trivially determined from Equation 11 as

$$\epsilon_{\min} = \tau_{\min}\frac{Q_{\text{in}}}{K} \tag{12}$$

This is the variation of pressure that occurs at maximum slope (either positive or negative) in time $\tau_{\min}$. As a result, the thresholds of the narrower working range are set as $p_{\max\_th} = p_{\max} - \epsilon_{\min}$ and $p_{\min\_th} = p_{\min} + \epsilon_{\min}$.

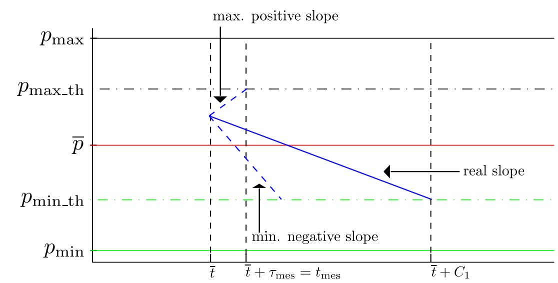


Fig. 5: Representation of the geometrical relationship between the quantities used in the calculation of τ_{mes}.

c) Evaluation of the sampling period: The sampling period τ_{mes} represents the variable time frame to trigger the next pressure measurement. It depends on the current inlet air flow, the current air pressure, and the values of inner thresholds. It is calculated in different states of the FSM. The calculation of τ_{mes} can be done using the following equation:

$$\tau_{\text{mes}} = \min\left(K\frac{p_{\max\_th} - p(t)}{q_{\text{in}}(t)}, K\frac{p_{\min\_th} - p(t)}{q_{\text{in}}(t) - Q_{\text{in}}}\right) \tag{13}$$

Equation 13 assumes that the evaluation is done at time t, when the system air pressure $p(t)$ is measured. If $\tau_{\text{mes}} < \tau_{\min}$ then τ_{mes} is set to $\tau_{\text{mes}} = \tau_{\min}$. The value of τ_{mes} is used to set the absolute time instant $t_{\text{mes}} = t + \tau_{\text{mes}}$ corresponding to the next measurement event (w.r.t. the current time t).

The terms within the $\min$ function in Equation 13 are obtained considering two worst cases, corresponding to the maximum and minimum slopes of the air pressure. Figure 5 shows the geometrical relationships between the involved values. Let's assume the inlet air flow has a known given value $q_{\text{in}}(t)$, which depends on $\Gamma^1(t)$. If the outlet air flow $q_{\text{out}}(t)$ has its maximum value, i.e., $q_{\text{out}}(t) = Q_{\text{out}}$, the air pressure variation is characterized by the highest negative slope. The worst case time is computed using Equation 10 by imposing $M = q_{\text{in}}(t) - Q_{\text{out}}$ and $p(t + \tau) = p_{min}$:

$$\tau = K\frac{p_{min} - p(t)}{q_{\text{in}}(t) - Q_{\text{out}}}$$

leading to the first term within the $\min$ function in Equation 13.

On the other hand, if $q_{\text{out}}(t) = 0$ the air pressure variation experiences the maximum positive slope. In this second situation, the worst case time is computed using Equation 10 by imposing $M = q_{\text{in}}(t)$ and $p(t + \tau) = p_{max}$, which leads to the second term in Equation 13:

$$\tau = K\frac{p_{max} - p(t)}{q_{\text{in}}(t)}$$

Notice that the value of τ_{mes} is always a finite number. In fact, the denominator of the two terms within the $\min$ function can not be both null at the same time. This is due to the fact that, from Equation 6, Q_{in} is a constant value strictly greater than zero. Therefore, the $\min$ function guarantees to consider the resulting finite term, in case the other term would be infinite.

V. DESCRIPTION OF THE FSM

This section describes in details the behavior of the control logic defined by the FSM of Figure 3 and by Algorithm 1. The control policy is encapsulated into 4 states of the FSM:

- `running`: corresponds to the regular operation of the multi-compressor;
- `measure`: the internal air pressure is measured and the next action is selected depending on the value of such measurement;
- `planning`: the configuration of compressors Γ^{01} is planned using Algorithm 1, and the load/unload of compressors in those sets is scheduled at a given time instant in the future, which is also computed in this phase;
- `recovery`: deals with possible violations of pressure operational constraints.

A. State `running`

While in the `running` state, the controller executes the control action previously planned and scheduled in some of remaining FSM states. Therefore, the current configuration of compressors $\Gamma^{01}(t)$ is maintained until a new scheduling event occurs. When the event takes place, the controller changes the configuration of compressors as planned. The system remains in the `running` state until the time reaches the value $t+\tau_{mes}$ or $t+\tau_{ctrl}$, whichever occurs first. In the former case, a new measurement is triggered, thus the next state is `measure`. In the latter case, a new control action need to be planned, therefore the next state is `planning`.

B. State `planning`

In the `planning` state, Algorithm 1 is applied to determine the next configuration of compressors and when such a configuration shall be enabled.

Due to the characteristics of the multi-compressor, which is made by a set of fixed speed compressors, it is not possible to achieve a constant air pressure in the system at any time. This is due to the discrete levels of inlet air flows determined by the combination of active compressors, which may not match exactly the amount of outlet air flow requested at a given time. To cope with this situation, the idea is to dynamically switch the compressors between the load and unload states, in order to balance the inlet and outlet air masses within a given time interval T. The time interval T can change between different control actions, since it depends on the actual system pressure, outlet and inlet air flows. In turn, this latter depends on which compressors are active during the period T.

The calculation of T is based on the observation that T can be divided into two consecutive time frames having duration C_1 and C_2, such as $C_1 + C_2 = T$. During the first interval (having duration C_1) the current configuration of compressors is maintained, i.e.

$$\Gamma^{01}(t) = \Gamma^{01}(\bar{t}) \quad \forall t \in [\bar{t}, \bar{t} + C_1]$$

being $\bar{t}$ the time instant when the planning action is performed, and $\bar{t} + C_1$ is the time instant at which the next control action will be triggered. The new configuration of compressors $\Gamma^{01}(\bar{t} + C_1)$ will be held until the time instant $\bar{t} + T$. In other words, the control algorithm applies the condition

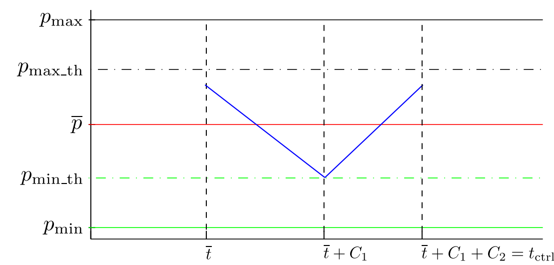


Fig. 6: Subdivision of the interval T in two time frame C_1 and C_2.

$$\Gamma^{01}(t) = \Gamma^{01}(\bar{t} + C_1) \quad \forall t \in [\bar{t} + C_1, \bar{t} + T].$$

Figure 6 shows an example of control action showing the subdivision of the time interval T.

The following calculations are developed to determine the value of C_1 and C_2, as well as other relevant values required by the algorithm. The calculations are explained in the reminder of this section, and properly referenced in the algorithm formulation.

First, the outlet air flow is evaluated on the basis of the estimation of the pressure derivative as follows:

$$q_{\text{out}}\left(\bar{t}\right) = q_{\text{in}}\left(\bar{t}\right) - \dot{p}\left(\bar{t}\right) K \tag{14}$$

where $\bar{t}$ is the current time instant and $\dot{p}\left(\bar{t}\right)$ derives from (9).

Since the outlet air flow profile can not be matched exactly due to the above considerations, the controller balances the input and output air mass.

The calculation starts by integrating Equation 2 between $\bar{t}$ and $\bar{t}+T$, thus obtaining the following relationship:

$$p(\bar{t}+T) - p(\bar{t}) = \frac{1}{K} \cdot \left\{ \int_{\bar{t}}^{\bar{t}+T} \dot{m}_{\text{in}}(t)\,\mathrm{d}t - \int_{t}^{t+T} \dot{m}_{\text{out}}(t)\,\mathrm{d}t \right\}$$

The balance of the inlet and outlet air masses implies the following condition:

$$\int_{\bar{t}}^{\bar{t}+T} \dot{m}_{\text{in}}(t)\,\mathrm{d}t = \int_{\bar{t}}^{\bar{t}+T} \dot{m}_{\text{out}}(t)\,\mathrm{d}t \tag{15}$$

This condition entails the regulation of the air pressure at the end of the considered time frame. In fact, Equation 15 brings to $p(\bar{t}+T) = p(\bar{t})$.

As previously anticipated, the T time interval is divided in two consecutive intervals having duration C_1 and C_2. During the first interval the inlet air mass is steadily maintained equal to the value assumed at the beginning of the interval (time $\bar{t}$). More formally

$$\dot{m}_{\text{in}}(t) = \dot{m}_{\text{in}}(\bar{t}) = q_{\text{in}}(\bar{t}) \quad \forall t \in [\bar{t}, \bar{t} + C_1]$$

This can be trivially obtained by keeping the current configuration of compressors $\Gamma^{01}(t)$ unchanged.

During the second interval, i.e. $[\bar{t} + C_1, \bar{t} + T]$, the inlet air flow is set to the value $\hat{Q}_{\text{in}}$. Moreover, we introduce the approximation that the outlet air flow remains constant within the same interval, i.e., $\dot{m}_{\text{out}}(t) = q_{\text{out}}(\bar{t})\ \forall t \in [\bar{t}, \bar{t}+C_1+C_2]$. Therefore, Equation 15 can be written as

$$\int_{\bar{t}}^{\bar{t}+C_1} q_{\text{in}}(\bar{t})\,\mathrm{d}t + \int_{\bar{t}+C_1}^{\bar{t}+C_1+C_2} \hat{Q}_{\text{in}}\,\mathrm{d}t = \int_{\bar{t}}^{\bar{t}+C_1+C_2} q_{\text{out}}(\bar{t})\,\mathrm{d}t$$

obtaining the equation to calculate C_2:

$$C_2 = C_1 \cdot \frac{q_{\text{in}}(\bar{t}) - q_{\text{out}}(\bar{t})}{q_{\text{out}}(\bar{t}) - \hat{Q}_{\text{in}}} \tag{16}$$

The specific control action to trigger depends on the sign of the air pressure derivative. In practice, different control actions are selected distinguishing among 3 different cases: increasing, decreasing or constant pressure.

d) Negative derivative: In case of negative derivative of the air pressure the controller has to turn on some compressors, since the pressure is decreasing due to the current air demand. The value of C_1 is calculated, being the duration of the time interval during which, without changing the set of active compressors, the pressure remains in the desired range. This is done by imposing $M = q_{\text{in}}\left(\bar{t}\right) - q_{\text{out}}\left(\bar{t}\right)$ and $p\left(\bar{t} + C_1\right) = p_{\text{min\_th}}$ in Equation 10:

$$C_1 = K \frac{p_{\text{min\_th}} - p\left(\bar{t}\right)}{q_{\text{in}}\left(\bar{t}\right) - q_{\text{out}}\left(\bar{t}\right)} \tag{17}$$

At this point, the controller has to select the set of compressors $\Gamma^1(\bar{t} + C_1)$ to switch to the load state. The selection must satisfy, for every compressor, the constraints on the maximum frequency of unload-to-load transitions specified by Equation 8. The condition is satisfied if the relationship $\bar{t} + C_1 \geq t_i^{\text{last\_on}}(\bar{t}) + \tau^{\text{delay}}$ holds. If a selected compressor does not satisfy the above constraint, the activation of that compressor is delayed. The activation time for that compressor is set to $t_i^{\text{last\_on}}(\bar{t}) + \tau^{\text{delay}}$. The adopted policy is to activate the compressors that are unloaded since the longest time. This is done to balance the utilization of compressors, thus improving their lifetime. The controller plans the new set $\Gamma^1(\bar{t}+C_1)$ of compressors to activate to achieve the condition $q_{\text{in}}(\bar{t} + C_1) \geq q_{\text{out}}\left(\bar{t}\right)$. This policy is intended to change the sign of the air pressure derivative at time $\bar{t} + C_1$, thus to determine the increase of the air pressure at that time instant. This part of the control logic is implemented by Algorithm 2, which is invoked at line 12 by Algorithm 1. After C_1 is determined, the value of C_2 is calculated by Equation 16.

The above calculations are valid in case $p(\bar{t})$ falls within the inner thresholds. Otherwise, the value of C_1 calculated with Equation 17 becomes negative, thus jeopardizing the controller logic. To deal with the case of $p_{\text{min}} \leq p(\bar{t}) \leq p_{\text{min\_th}}$, C_1 is set to zero at line 8, while C_2 is set equal to the time required to reach $p_{\text{max\_th}}$. i.e.

$$C_2 = K \frac{p_{\text{max\_th}} - p\left(\bar{t}\right)}{q_{\text{in}}\left(\bar{t}\right) - q_{\text{out}}\left(\bar{t}\right)} \tag{18}$$

This ensures $C_2 > 0$, thus guaranteeing the correct behavior of the control logic.

e) Positive derivative: In case of positive derivative the inlet air flow is higher than the outlet flow. Therefore, the controller has to unload some compressors to compensate the extra inlet flow. Similarly to the previous case, the controller calculates the duration of the time interval during which, if the set of active compressors is not changed, the pressure remains in the desired range.

This is done by imposing $M = q_{\text{in}}\left(\bar{t}\right) - q_{\text{out}}\left(\bar{t}\right)$ and $p\left(\bar{t} + C_1\right) = p_{\text{max\_th}}$ in Equation 10:

$$C_1 = K \frac{p_{\text{max\_th}} - p\left(\bar{t}\right)}{q_{\text{in}}\left(\bar{t}\right) - q_{\text{out}}\left(\bar{t}\right)} \tag{19}$$

The compressors to unload are selected among those having the the longest activation time. Again, this is done to balance the utilization of compressors. Moreover, in this specific case, this policy allows to increase the duration of the time frame between two consecutive unload-to-load transitions for the considered compressors. The controller plans the new set $\Gamma^0(\bar{t} + C_1)$ of compressors to unload to achieve the condition $q_{\text{in}}(\bar{t} + C_1) \leq q_{\text{out}}\left(\bar{t}\right)$. In this way, the air pressure at time $\bar{t} + C_1$ is planned to decrease. This part of the control logic is described by Algorithm 3, which is invoked at line 26 by Algorithm 1. Finally, the value of C_2 is calculated by Equation 16.

Again, the calculations hold in case $p(\bar{t})$ falls within the inner thresholds. Otherwise, the value of C_1 calculated with Equation 19 becomes negative. To deal with the case of $p_{\text{max}} \geq p(\bar{t}) \geq p_{\text{max\_th}}$, C_1 is set to 0 at line 22, while C_2 is set equal to the time required to reach $p_{\text{min\_th}}$. i.e.

$$C_2 = K \frac{p_{\text{min\_th}} - p\left(\bar{t}\right)}{q_{\text{in}}\left(\bar{t}\right) - q_{\text{out}}\left(\bar{t}\right)} \tag{20}$$

f) Null derivative: In this case there is no variation of the air pressure. The obvious policy is to maintain the current configuration of compressors $\Gamma^{01}(t)$ unchanged, since they are supplying the exact amount of air mass to match the demand. Therefore, τ_{ctrl} is set equal to τ_{mes} (see Equation 13). This is equivalent to set C_1 equal to τ_{mes} and $C_2 = 0$, i.e., $T = \tau_{\text{mes}}$.

C. State `measure`

In the `measure` state, the air pressure is sampled. Let's assume to perform this action at time $\bar{t}$. If the pressure is outside the working range $[p_{\text{min}}, p_{\text{max}}]$ the `recovery` state is triggered. Otherwise, the outlet air flow $q_{\text{out}}(\bar{t})$ is estimated using Equation 14. If the estimated value did not changed after the previous control action, τ_{mes} is set according to

Algorithm 1 Algorithm executed during the `planning` state.

calculate $\dot{p}\left(\bar{t}\right)$
calculate $q_{\text{out}}(t)$ (Equation 14)
$\hat{Q}_{\text{in}} = q_{\text{in}}(t)$
if $\dot{p}\left(\bar{t}\right) = 0$ **then**
  set τ_{mes} (Equation 13)
else if $\dot{p}\left(\bar{t}\right) < 0$ **then**
  if $p(\bar{t}) \leq p_{\text{min\_th}}$ **then**
    $C_1 = 0$
  else
    calculate C_1 (Equation 17)
  end if
  determine $\Gamma^{01}(\bar{t} + C_1)$ using Algorithm 2
  if $p(\bar{t}) \leq p_{\text{min\_th}}$ **then**
    calculate C_2 as in Equation 18
  else
    calculate C_2 (Equation 16)
  end if
  set $\tau_{\text{ctrl}} = \bar{t} + C_1 + C_2$
  set τ_{mes} (Equation 13)
else
  if $p(\bar{t}) \geq p_{\text{max\_th}}$ **then**
    $C_1 = 0$
  else
    calculate C_1 (Equation 19)
  end if
  determine $\Gamma^{01}(\bar{t} + C_1)$ using Algorithm 3
  if $p(\bar{t}) \geq p_{\text{max\_th}}$ **then**
    calculate C_2 as in Equation 20
  else
    calculate C_2 (Equation 16)
  end if
  set $\tau_{\text{ctrl}} = \bar{t} + C_1 + C_2$
  set τ_{mes} (Equation 13)
end if

Algorithm 2 Algorithm to select the compressors to activate in case of negative air pressure derivative.

sort Γ^0 in ascending order of $t_i^{\text{last\_on}}(\bar{t})$
$i = 0$
while $\hat{Q}_{\text{in}} < q_{\text{out}}(t)$ **do**
  get the c_i compressor from Γ^0
  if $\bar{t} + C_1 \geq t_i^{\text{last\_on}}(\bar{t}) + \tau^{\text{delay}}$ **then**
    $t^{\text{on}} = \bar{t} + C_1$
  else
    $t^{\text{on}} = t_i^{\text{last\_on}}(\bar{t}) + \tau^{\text{delay}}$
  end if
  set $s_i(t^{\text{on}}) = 1$
  $\hat{Q}_{\text{in}} = \hat{Q}_{\text{in}} + Q_i$
  $i = i + 1$
end while

Equation 13; the configuration of compressors is not affected, and the FSM is driven towards the `running` state. Otherwise, the `planning` state is entered.

Figure 7 shows an example of this condition. The controller detects the change of outlet air flow in the `measure` state and the FSM is driven towards the `planning` state. In this state the proper control action is selected to manage the new outlet air flow.

D. State `recovery`

This state is entered from the `measure` when the pressure falls out the desired working range $[p_{\min}, p_{\max}]$. This state is also used to start the system, when the internal pressure is null, in order to enter the normal operation range. The control

Algorithm 3 Algorithm to select the compressors to unload in case of positive air pressure derivative.

sort $\Gamma^1(\bar{t})$ in ascending order of $t_i^{\text{last\_on}}(\bar{t})$
$i = 0$
while $\hat{Q}_{\text{in}} < q_{\text{out}}(t)$ **do**
  get the c_i compressor from $\Gamma^1(\bar{t})$
  set $s_i(\bar{t} + C_1) = 0$
  $\hat{Q}_{\text{in}} = \hat{Q}_{\text{in}} - Q_i$
  $i = i + 1$
end while

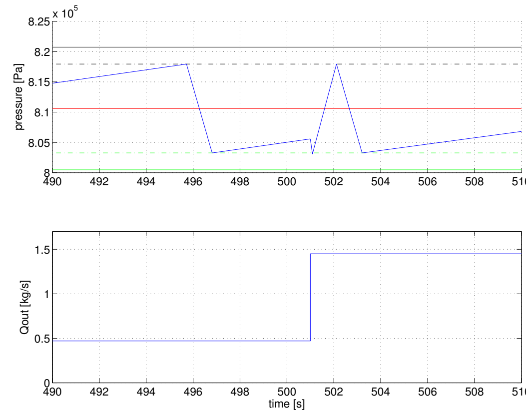


Fig. 7: Representation of the system behavior upon a change of the outlet air flow $q_{\text{out}}(t)$.

action triggered in this state is conceived to reach the set-point as quickly as possible. If $p(\bar{t}) < p_{\min}$, all compressors whose activation guarantees the satisfaction of Equation 8, are simultaneously activated. If instead $p(\bar{t}) > p_{\max}$, all compressors are unloaded. The next control action is scheduled at time $t_{\text{ctrl}} = \bar{t} + \tau_{\text{ctrl}}$, where

$$\tau_{\text{ctrl}} = K \frac{\left|\overline{p} - p\left(\bar{t}\right)\right|}{\left|q_{\text{in}}(\bar{t}^{+}) - q_{\text{out}}\left(\bar{t}\right)\right|} \tag{21}$$

In Equation 21, the term $q_{\text{in}}(\bar{t}^{+})$ corresponds to the inlet air flow generated after the new configuration of active compressors is set at time $\bar{t}$. This is the time required to bring the system to set-point $\overline{p}$ from the current pressure $p\left(\bar{t}\right)$. Equation 21 is obtained from Equation 10 by setting $M = q_{\text{in}}(\bar{t}^{+}) - q_{\text{out}}\left(\bar{t}\right)$.

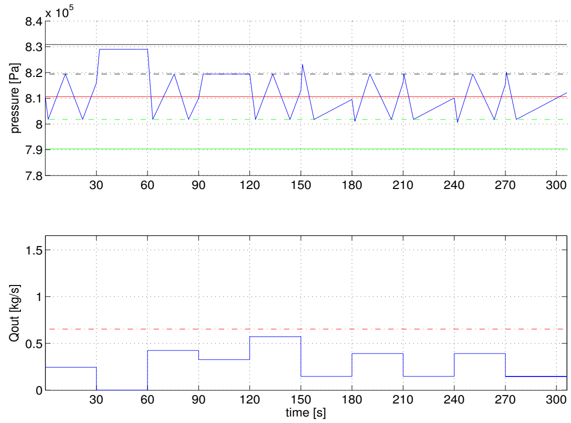
In this state the timer t_{mes} is not set. This avoids a worthless transition towards the `measure` state during the time frame $[\bar{t}, \bar{t} + \tau_{\text{ctrl}}]$. In fact, if the pressure is still outside the working range, the `measure` state will trigger another transition to the `recovery`, and the same condition would be managed twice.

VI. Simulation Results

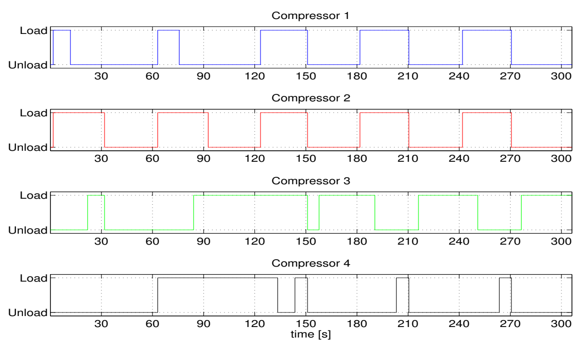
This section provides some simulation evaluations of the control algorithm behavior. Simulation parameters are listed in Table I. In the simulations, the air requested from the loads is piecewise constant. The simulations show – from top to bottom

TABLE I: Values of parameters used in the simulation.

Parameter	Symbol	Value
Number of compressors	N	4
Set-point pressure	$\overline{p}$	810 600 Pa
Pressure tolerance	ϵ_p	20 265 Pa
System volume	V	$4\,\mathrm{m}^3$
Air temperature	Θ	298,15 K $\equiv$ 25 °C
Minimum time between two compressor activations	τ^{delay}	10 s
Inlet air mass flow of each compressor	Q_{i}	0,163 kg/s
Sampling period lower bound	τ_{min}	0,82 s

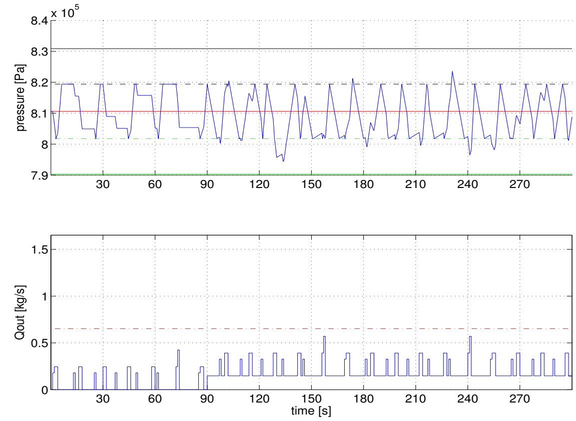


(a)

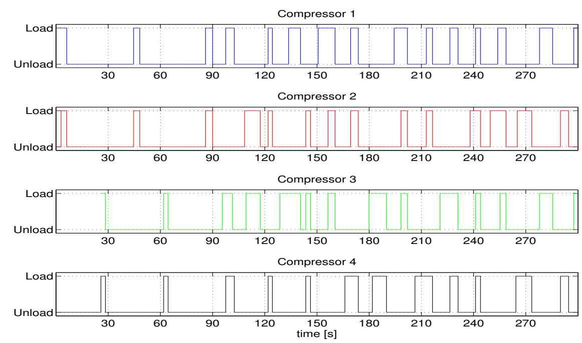


(b)

Fig. 8: Simulation of the multi-comp system with a low frequency $q_{\text{out}}(t)$ profile. 8a shows the air pressure (top half) and the outlet air profile (bottom half). 8b shows the schedule of compressors.



(a)



(b)

Fig. 9: Simulation of the multi-comp system with a high frequency $q_{\text{out}}(t)$ profile. 9a shows the air pressure (top half) and the outlet air profile (bottom half). 9b shows the schedule of compressors.

– the air pressure profile, the outlet air flow profile, and the schedule generated for the compressors.

In Figure 8 the outlet air flow $q_{\text{out}}(t)$ has a relatively low transition rate, with transitions occurring every 30 seconds. Notice that in the time intervals $[30, 60]$ and $[90, 120]$ (seconds) the slope of the air pressure is null. In the former case, the controller unloads all the active compressors since the value of $q_{\text{out}}(t)$ is null. It is worth to note that during this time frame the air pressure is above the upper inner threshold. This is caused by the sudden decrease of air demand happening at $t = 30$, which is detected at the next measurement event. The delay between the change and the detection causes the air pressure to exceed the upper inner threshold. However, the condition of Equation 12 on ϵ_{min} guarantees to detect the variation before exceeding the working range. In the latter case, the null slope is due to the exact match between the outlet air flow $q_{\text{out}}(t)$ and the inlet air flow, equal to $2Q_i$. In other words, the outlet air flow can be matched exactly by activating 2 compressors.

The dynamics of the outlet air flow requires an average period between consecutive unload-to-load transitions of around 61 seconds, with a minimum time frame of about 58 seconds. In this case, therefore, the constraint on the minimum period between transitions, which is set to 10 seconds, is largely guaranteed.

A second test-case is simulated imposing faster dynamics to the loads. Different loads have different air demand duty cycles, whose combination generates the overall outlet air flow profile. Let's describe the activation pattern of the i-th load with (P_i, C_i, ϕ_i) [seconds], where T_i is the period of the activation, C_i is the time the load is active in each period, and ϕ_i is the initial offset. Three loads are considered having the following timing parameters: $(12, 1, 1)$, $(14, 2, 2)$, $(300, 210, 90)$. Figure 9 shows the patterns generated under the above conditions. Despite the outlet air flow changes more

TABLE II: Statistical evaluation of working range violations.

quantity	value	unit
avg duration of a violation	42.18	[sec]
avg amount of a violation	6484	[Pascal]
avg # violations per simulation	1.13	–
simulations with violations	36.1	%
time in violation	5.73	%

frequently, the controller is able to maintain the air pressure within the range $[p_{\text{min\_th}}, p_{\text{max\_th}}]$. Moreover, the slope of the pressure is always non-null, since the outlet air flow never has a value being integer multiple of Q_i. The average time frame between two consecutive activations is 21 seconds, with a minimum of such value of around 11 seconds.

These values suggest that the multi-compressor becomes more stressed – in terms of activation frequency of compressors – when loads with higher duty-cycles are present. Our simulations have shown a relevant sensitivity of the air pressure regulation performance w.r.t. the activation period of loads. The problem with fast periods is that, due to the constraint specified by τ^{delay} (Equation 8), one or more compressors may be prevented to switch from unload to load since the last transition was happened too recently. Therefore, the compressors that should be switched on must be delayed to avoid overheating, and the pressure may fall below the lower working threshold.

Extended simulations were performed to assess the behavior of the multi-compressor under the above circumstances. A total of 1000 simulation runs were performed, each one lasting 300 seconds. In each run, a variable random number of loads between 4 and 8 were selected (uniform discrete distribution). The period of each load was selected with uniform distribution $T_i \sim U(10, 60)$ seconds. The duty-cycle D_i of the load was randomly generated as $D_i \sim U(0.1, 0.8)$, while the activation time was determined as $C_i = D_i \cdot T_i$ [seconds]. In these simulations, we have evaluated the characteristics of violations, where a violation corresponds to the air pressure falling below the lower threshold of the working range. The following quantities were evaluated: the number of violations; the average duration of violations; the average number of violations per simulation; the number of runs containing a violation; the amount of violation. The amount of violation is calculated as $p_{\min} - \min p(t)$ over the entire run. The average amount of violation is calculated on these values. Overall, the amount of violations is limited w.r.t. the working range (15% in average). On the other hand, the total time of the system working under violation conditions is equal to 5.73% of the total simulation time. These values are summarized in Table II.

VII. Conclusion and Future Work

This paper described the design of the controller for a multi-compressor system. Design goals are the coordination of the set of compressors composing the unit to achieve adequate performance in terms of air pressure regulation. A key issue was related to the limitation of the frequency of unload-to-load transitions, to protect the device components from overheating. The dissertation focused on the derivation of timing constraints related to key control events.

The planned future work will deal with the derivation of further analytical results about the system behavior. Moreover, experimental tests are planned on a real industrial setup. The experiments will assess the system behavior under real working conditions. The design of the control software will include the mapping of timing constraints on a set of real-time tasks and the implementation of the control algorithm on top of a real-time operating system.

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