

# Real-Time Modeling and Control of a Cyber-Physical Energy System

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**Abstract**—This paper introduces an approach for applying real-time scheduling techniques to balance electric loads in cyber-physical energy systems. The proposed methodology aims to determine, guarantee and optimize an upper bound on the peak load of electric power, which represents a desirable feature for both the electricity supplier and the user of the electrical system. For this purpose, networked electric devices are modeled using parameters derived from the real-time scheduling discipline used for computing systems. Therefore, the upper bound can be enforced by predictably and timely switching on/off the electric devices composing the electrical system.

The paper contribution include: the illustration of the relevance of electric load balancing in cyber-physical energy systems, motivating the use of real-time scheduling techniques to achieve predictability of electric loads scheduling; the presentation of a novel and powerful modeling methodology of the physical system based on a set of periodically activated loads, to enable the use of traditional real-time system models and scheduling algorithms, with adequate adaptations, to manage loads activation/deactivation. We finally derive interesting properties of real-time parameters and provide theoretical results concerning the computation of their values.

**Keywords**-Cyber-Physical Energy Systems; Load Balancing; Real-Time; Modeling; Peak Load

## I. INTRODUCTION

The use of embedded systems to automatically monitor and control networks of electric devices is growing in interest in both home and industrial domains. In industrial applications, embedded systems have been traditionally employed to automate industrial processes. In many applications, real-time constraints arise from the physical process and must be guaranteed by computing tasks in order to meet the application requirements. Conversely, in building automation (domotics) the integration of embedded devices with domestic appliances is bringing to the possibility of implementing a large amount of new features and to advance the performance of existing ones, including multimedia, security, and energy efficiency.

In recent years, the research on embedded systems is moving towards the integration of computational resources within the physical system under monitoring and control, leading to the so-called *Cyber-Physical Systems* (CPSs), i.e., complex networks of interconnected embedded devices

tightly integrated with the physical process under control. Key issues in CPSs are sensing and actuation, the modeling of the physical system, real-time computing, and networking. Challenging issues arise from the dynamic nature of the underlying physical process, that requires the ability of working under uncertain environmental conditions, and involve complex relationships among an high number of system components. Example applications for CPSs are in the field of manufacturing control, energy systems, automotive and avionics systems, traffic control, medical systems, cooperative robotics and smart buildings. Some detailed examples of CPSs can be found in [1].

On the other hand, home and industrial automation systems more and more often require to address the issue of energy efficiency. For this purpose, the research on CPSs has been extended to the study of *Cyber-Physical Energy Systems* (CPESs) [2]. In those systems, embedded computing is integrated within the electrical system to gather information about the most important electric parameters, such as voltage, current, phases, consumed energy and power. Environmental parameters, as temperature, umidity and pressure, are also relevant for the system characterization. The acquired data are then combined and processed to generate suitable control commands for the electric devices, in order to achieve the desired application goal. And such goals include, or introduce constraints, on power and energy usage.

In cyber-physical energy systems, as addressed in this paper, the physical process is composed by a set of electric devices that are monitored and suitably operated by a set of networked embedded systems. A relevant example of energy system automation is represented by the so-called *smart grid* [3]. Smart grids focus on the interaction between energy supply and usage, in which a two-way flow of electricity between energy providers and users is supported by a pervasive, distributed and interconnected information infrastructure. A fundamental role is played by smart meters for the intelligent monitoring of buildings, districts and town energy usage [4]. Renewable source of energy are also an important part of a smart grid, as well as distributed power generation systems by means of micro-cogeneration systems (or Combined Heat and Power, CHP, systems).

The availability of compact and flexible embedded sys-

tems allows the effective implementation of cyber-physical energy systems. Monitoring tasks and control actions can be applied on devices composing the considered physical system. Moreover, the involved embedded systems can be connected to build large distributed control networks.

This paper focuses on cyber-physical energy systems dedicated to electric power management, and particular attention is devoted to the balancing of power usage, which represents an important issue in electrical systems [5]. Balancing the use of power aims at avoiding dangerous peak load conditions, i.e., when too many user devices are simultaneously active, with the risk of overloads on the power distribution infrastructure leading to possible blackouts. The main goals of the proposed approach is to guarantee that the peak power demand remains under a given threshold, and to determine a smoother and flatter curve of power usage over time.

The proper management of peak load conditions is desirable by both consumers and energy providers [6]. The supplier can achieve a better balance over its distribution infrastructure and a tighter design of the system (e.g., optimizing the size of cables). For example, energy providers have the possibility to turn off the least efficient (i.e. the most expensive) power plants while achieving their contractual provisions; moreover, they can build the electric distribution infrastructure for tolerating well defined (and possibly, reduced) peak loads, with limited technical issues and reduced costs. On the other hand, consumers can negotiate better pricing conditions if they can guarantee an upper bound on power usage, considering that pricing strategies are often driven by policies such as *peak-load pricing* [7]. Additionally, energy management can be integrated with smart metering devices to increase the awareness of users about energy issues generated by their habits [8] which, in turn, can drive the behavioural change widely recognized as an important component of energy saving strategies.

The methodology proposed in this paper aims to enforce the guarantee on the peak load of power usage by adequately control the available set of electrical devices composing the physical system in a cyber-physical energy system. More precisely, the idea is to schedule the activation of devices in a timely and predictable manner, in order to limit the concurrent activation of loads, thus balancing the total consumed electric power and reducing the peak load of power usage. Moreover, it provides the possibility to determine the value of the peak load in worst case conditions; such information is important to realize a tight design of the energy distribution system (e.g., the size of cables), avoiding undesired extra-costs.

The physical system will be properly modeled to apply scheduling algorithms suitably derived from the real-time scheduling discipline which is currently developed for computing tasks executed on a microprocessor (see [9] for an introduction and a comprehensive description of hard real-time systems). One inherent benefit of this approach

is to take advantage of the strong mathematical background which characterizes the results of real-time scheduling analysis. Moreover, the powerful analysis techniques developed over more than three decades of research on real-time systems will be leveraged to characterize timing and energy properties of the physical system, while facing the problem of dealing with large and complex systems, with several types of constraints, which is a typical scenario when cyber-physical energy systems are considered.

This innovative approach to electric load management opens the door to the application of sophisticated scheduling algorithms to meet power, energy and timing constraints in cyber-physical energy systems.

#### A. Paper organization

The paper is organized as follows: Section II illustrates the approach to model an electrical system as a set of periodically activated loads, and in Section III some relevant related works are recalled and commented. Section IV introduces the system model adopted in this work, while an example of physical process that can be represented with such a model is shown in Section VI. Section V discusses how to associate suitable real-time parameters to the physical process, and derives some interesting properties. The obtained theoretical results are applied to an example test case in Section VII. Finally, Section VIII states our conclusions and provides a sketch of the several possible enhancements to this work.

## II. TOWARDS MODELING ELECTRIC LOADS USING REAL-TIME PARAMETERS

The main contribution of this paper is to propose an approach to model electric devices using real-time parameters. For this purpose, we establish an analogy between real-time computing systems and electrical systems, which represent the physical background of cyber-physical energy systems.

Real-time computing systems are used to allow the concurrent execution of processing tasks subject to timing constraints on a processor. However, in more general terms, the real-time scheduling problem can be defined as the problem of allocating resources over time to a set of time-consuming tasks, while meeting a given set of timing constraints. The key observation is that, under this definition, resources may not necessarily be processors or processing devices, as they are usually intended in computing systems.

In the last few years, the use of real-time scheduling techniques has been extrapolated from the field of computing system to be used in different application domains. In communication systems, real-time algorithms are applied to schedule and analyse the performance of messages sets over a communication channel (e.g., [10]). In this case, an analogy is made between computing and communication systems, where messages are made equal to computing tasks. The real-time scheduling is thus performed, respectively, on the communication channel and processors. In processing

systems, timing constraints need to be guaranteed on the execution times, while they must be achieved on message's end-to-end latency in communication systems. This means that a real-time task must be guaranteed to terminate its execution before its deadline, while a message must be delivered to the receiver within the given time limit. The analogy between computing and communication systems allows to extend interesting results from one domain to the other, and vice-versa. An example of technique originally proposed for the modeling and the analysis of communication networks that has been successfully adapted to real-time computing systems is the *network calculus*, a theoretical framework that allows the analysis of information flow in computer networks subject to several kind of constraints [11]. Such adaptation led to the so-called *real-time calculus* [12]).

The above considerations foster the possibility of using real-time scheduling techniques to model, to analyse and to manage technological systems that would present a sufficient degree of affinity. Following this approach, the proposed methodology establishes an analogy between electric loads and computing tasks subject to real-time constraints.

In this paper, we borrow the well-known periodic task model [13] widely studied in real-time systems to represent electric loads as periodically triggered activities. A bound is imposed on the total amount of time that a load can stay active in each period. As for computing tasks, all the time properties of electric loads (periods, deadlines and activation time) must be selected according to their application requirements. The activation time plays the role of task's WCET (Worst Case Execution Time) in real-time computing systems. We assume that a load consumes a given amount of electric power while active, and no power when switched off. Based on such a model, a priority-based scheduling algorithm can be applied to selectively activate/deactivate devices. The goal is to meet the timing constraints of each load, while guaranteeing an upper bound on the total instantaneous power consumed by the concurrent activation of electric devices.

### III. RELATED WORKS

Power-aware scheduling techniques represent an active research topic in the real-time systems literature. The typical approach behind such techniques is to exploit some dedicated features of modern electronic components (micro-processors, motherboards, etc.) to reduce the total amount of energy consumption. For example, the Dynamic Voltage Scaling (DVS) [14] technique, i.e., the possibility of dynamically changing the power supply voltage of a microprocessor, is leveraged to reduce the energy consumption of the processing unit. Since reducing the supply voltage brings to a decrease of the clock speed, the analysis focuses on the guarantee of real-time constraints when processor's speed is allowed to change. However, the goal of those techniques is

to reduce the total use of energy, while our objective is to reduce the peak load of electric power.

Other approaches are more related with energy systems. In [15], the authors aim to find optimal schedules for microCHP (Combined Heat and Power) systems using a global optimization technique based on an Integer Linear Programming formulation of the problem. This is essentially an off-line approach, while on-line scheduling is addressed in this paper. Some recent works are concentrating on the real-time issues related with batteries charge/discharge, for example in electric vehicles [16].

In [17] the authors tackle the problem of scheduling tasks on a system powered by the energy generated from renewable sources. The goal is to produce a suitable schedule to maintain the battery energy within a predefined range. A dedicated algorithm is proposed, that uses a closed loop approach to track the change in the available energy and to adapt the schedule accordingly. Our approach, instead, uses an open loop technique to obtain the same goal, despite we do not restrict our target application to batteries. It is worth to note that closed-loop techniques are inherently more robust to noise and errors. Actually, we are planning to suitably integrate feedback techniques on top of our model as future work.

### IV. SYSTEM MODEL

The considered system is composed by a set  $\Lambda = \{\lambda_1, \dots, \lambda_n\}$  of  $n$  loads that need to be periodically turned on and off (or activated/deactivated), depending on their specific timing constraints. A load is said to be *active* when it is turned on, *inactive* otherwise. The load activity is controlled by a *load scheduler* that decides when each load is activated/deactivated. The activation of each load is independent of other loads (i.e., no precedence or other kind of constraints among loads are considered). Formally, the scheduler assigns to each load  $\lambda_i$  a schedule that is modeled by the function  $s_i(t)$ :

$$s_i(t) = \begin{cases} 1 & \lambda_i \text{ is active at } t \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The schedule of all loads is then given by  $s(t) = \{s_1(t), \dots, s_n(t)\}$ . The schedule will be denoted with  $\mathcal{S}$  when the dependency from the time is not relevant.

The load  $\lambda_i$  consumes a  $p_i(t)$  amount of electric power when active at time  $t$ , no power otherwise. More formally, it holds

$$p_i(t) = \begin{cases} P_i, & \text{if } s_i(t) = 1 \\ 0, & \text{if } s_i(t) = 0 \end{cases}$$

In this paper, we focus the goal of our approach on the control loads characterized by time-varying state variable  $x_i(t)$ . The notation  $x(t) = (x_1(t), \dots, x_n(t))$  will be used to denote the state vector representing all loads. The state

vector value varies over time with the law specified by Equation 2.

$$\begin{aligned} \dot{x}(t) &= \rho(t) \\ x(0) &= \bar{x} \end{aligned} \quad (2)$$

where

$$\rho(t) = \rho^{\text{in}} - \rho^{\text{out}} s(t) \quad (3)$$

$$0 < \rho^{\text{in}} < \rho^{\text{out}} \quad (4)$$

In other words, each state variable  $x_i$  linearly increases with a slope defined by  $\rho_i^{\text{in}}$  when  $\lambda_i$  is inactive (i.e.,  $s_i = 0$ ), while it linearly decreases with slope defined by  $\rho_i^{\text{in}} - \rho_i^{\text{out}}$ , which is lower than 0 due to inequality 4, when  $\lambda_i$  is active. Notice that the choice of associating a decreasing state variable with an active load and viceversa does not affect the generality of the problem statement and its solution.

#### A. Real-time modeling

Considering the parameters used to model a traditional real-time computing task, we use the tuple  $\{T_i, C_i, P_i\}$  to define a load  $\lambda_i$ , where

- $T_i$  is the time frame between two consecutive activations (as in the periodic task model for real-time computing tasks [13]);
- $C_i (\leq T_i)$  represents the activation time duration of  $\lambda_i$  within each period  $T_i$ ;
- $P_i$  is the nominal power associated to the activation of  $\lambda_i$ , as previously stated.

The utilization of  $\lambda_i$  is defined as

$$U_i = \frac{C_i}{T_i} \quad (5)$$

while the total utilization of the load set is  $U = \sum_{i=1}^n U_i$ .

The  $k$ -th request for activating the load  $\lambda_i$  happens at time  $r_{i,k}$  (request time), where  $r_{i,k} = kT_i$ ,  $k \in \mathbb{N}$ .

*Definition 1:* A schedule  $\mathcal{S}$  is said to be *valid* if it assigns to each load  $\lambda_i$  an amount of activity time equal to  $C_i$  between two consecutive request times. Formally,

$$\forall \lambda_i, \forall k \quad \int_{r_{i,k}}^{r_{i,k+1}} s_i(t) dt = C_i \quad (6)$$

A valid schedule can be generated by a real-time scheduling algorithm as as Earliest Deadline First (EDF) or Rate Monotonic (RM) [13] when applied to a feasible set of loads, i.e., the specific schedulability test, applied to the given load set, is passed for the considered algorithm. Since we are considering implicit deadlines, i.e., deadlines equal to periods, utilization-based schedulability tests can be used for both algorithms.

*Definition 2:* The overall instantaneous power consumption  $p(t)$  is defined as

$$p(t) = \sum_{i=1}^n p_i(t). \quad (7)$$

*Definition 3:* The *peak load*  $P$  of a set of loads is defined as the maximum instantaneous power consumption over the system lifetime:

$$P = \max_{t \geq 0} p(t). \quad (8)$$

#### B. Problem statement

Given the system model and the possibility of adequately modeling the involved electric loads using real-time parameters, we are interested to determine the relationship between the physical system parameters (i.e.,  $\rho^{\text{in}}$ ,  $\rho^{\text{out}}$ , and  $\bar{x}$ ) and real-time parameters (basically,  $T_i$  and  $C_i$ ) such that the overall peak load  $P$  is minimized while meeting two constraints: i) the scheduler will produce a valid schedule (see Definition 1); ii) the instantaneous state variable value  $x_i(t)$  of each load  $\lambda_i$  is bounded in the range  $[x_i^{\text{min}}, x_i^{\text{max}}]$ . More formally,

$$\begin{aligned} &\text{minimize } P \\ &\text{such that } \begin{cases} \mathcal{S} \text{ is a valid schedule} \\ \forall i, x_i^{\text{min}} \leq x_i(t) \leq x_i^{\text{max}} \end{cases} \end{aligned} \quad (9)$$

In the remainder of this paper, we will also use the compact vector notation

$$x^{\text{min}} = (x_1^{\text{min}}, \dots, x_n^{\text{min}})$$

and a similar notation will be used to indicate  $x^{\text{max}}$ .

### V. PROPERTIES AND RESULTS OF PHYSICAL AND REAL-TIME PARAMETERS

In this section we determine some interesting properties and introduce relevant results regarding the calculation of real-time parameters of the system model presented in Section IV.

First, we establish a relationship between the load utilization  $U_i$  and the dynamical properties of the related physical process. This relationship determine an useful property of the state variable itself.

*Theorem 1:* If the utilization  $U_i$  of task  $\lambda_i$  is set as

$$U_i = \frac{\rho_i^{\text{in}}}{\rho_i^{\text{out}}} \quad (10)$$

then the state variable assumes the same value  $\bar{x}_i$  at every request time  $r_{i,k}$ , i.e.,

$$\forall k \in \mathbb{N} : k \geq 0 \rightarrow x_i(kT_i) = \bar{x}_i. \quad (11)$$

*Proof:* We start by integrating Equation 2 to obtain the state variable value at the generic  $k$ -th request time  $r_{i,k}$ :

$$x_i(kT_i) = x_i(0) + \int_0^{kT_i} \rho_i(t) dt \quad (12)$$

Considering the definition of  $\rho(t)$  (Equation 3), Equation 12 can be rewritten as

$$x_i(kT_i) = \bar{x}_i + \rho_i^{\text{in}} \int_0^{kT_i} dt - \rho_i^{\text{out}} \int_0^{kT_i} s_i(t) dt \quad (13)$$

While the value of the first integral is trivial, the second integral can be easily evaluated by considering the definition of valid schedule (Equation 6) and the definition of task utilization  $U_i$ :

$$\begin{aligned} x_i(kT_i) &= \bar{x}_i + \rho_i^{\text{in}} kT_i - \rho_i^{\text{out}} U_i kT_i \\ &= \bar{x}_i + kT_i (\rho_i^{\text{in}} - \rho_i^{\text{out}} U_i) \end{aligned} \quad (14)$$

Finally, introducing the term  $U_i$  from Equation 10 into Equation 14, it follows

$$x_i(kT_i) = \bar{x}_i$$

which proves the theorem.  $\blacksquare$

Theorem 1 states that, to achieve the result specified by Equation 11, the load utilization  $U_i$  depends only on  $\rho^{\text{in}}$  and  $\rho^{\text{out}}$ . In particular, it does not depend on the state variable range bounds. Moreover, since a state variable assumes the same value at every request time, we are allowed to derive global properties of state variable behaviour by analyzing such behaviour within the time frame delimited by one period.

Since we are interested to bound the state variable variation within a specified range, we introduce the definition of largest variation with respect to  $\bar{x}_i$ :

*Definition 4:* We define the largest ascending and descending variations of  $x_i(t)$  with respect to  $\bar{x}_i$ , respectively, as follows:

$$\Delta_i^{\text{inc}} \equiv \left( \max_t x_i(t) \right) - \bar{x}_i \quad (15)$$

$$\Delta_i^{\text{dec}} \equiv \bar{x}_i - \min_t x_i(t) \quad (16)$$

Notice that definitions (15) and (16) are calculated for every  $t$ , and thus represent a global behaviour.

We now determine the properties of state variable variations within one time period.

*Lemma 1:* The largest possible ascending variation of the state variable  $x_i(t)$  with respect to  $\bar{x}_i$  on a period is

$$\delta_i^{\text{inc}} = \rho_i^{\text{in}} (T_i - C_i) \quad (17)$$

*Proof:*

Let us define  $\hat{t}$  as the time instant after which the state variable  $x_i(t)$  can only decrease, i.e.,

$$\forall t : \hat{t} < t \leq T_i \rightarrow x_i(t) < x_i(\hat{t})$$

Therefore, the maximum value of  $x_i(t)$  must correspond to a time instant  $t^*$  such as  $0 < t^* \leq \hat{t}$ . The value of  $x_i(\hat{t})$  can be calculated by integrating Equation 2, obtaining

$$x_i(\hat{t}) = x_i(0) + \int_0^{\hat{t}} \rho_i(t) dt \quad (18)$$

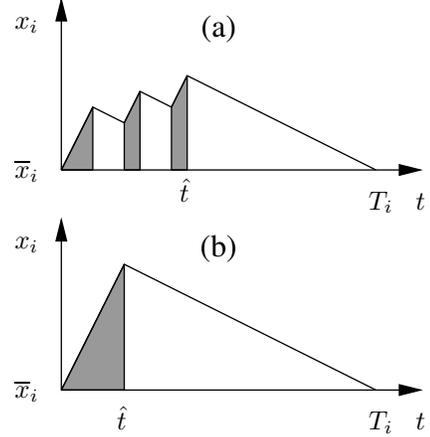


Figure 1. Examples of behaviour of the state variable variation within a period  $T_i$ ; figure (b), in particular, represents the case when the state variable's increasing variation is maximized.

which can be rewritten as

$$x_i(\hat{t}) = \bar{x}_i + \rho_i^{\text{in}} \int_0^{\hat{t}} dt - \rho_i^{\text{out}} \int_0^{\hat{t}} s_i(t) dt \quad (19)$$

The first integral corresponds to the amount of time that  $x_i(t)$  increases in the range  $[0, \hat{t}]$ , which is equal to  $T_i - C_i$ . In fact, by definition of  $\hat{t}$ , the range  $[0, \hat{t}]$  contains all the amount of time that the state variable has a negative derivative and, since  $C_i$  is the amount of time that the state variable has a positive derivative in the whole range  $[0, T_i]$ , then the amount of time that the state variable derivative is negative equals  $T_i - C_i$ . Therefore

$$x_i(\hat{t}) = \bar{x}_i + \rho_i^{\text{in}} (T_i - C_i) - \rho_i^{\text{out}} \int_0^{\hat{t}} s_i(t) dt \quad (20)$$

The proof concludes by noticing that Equation 20 is composed by two constants and a negative term (the value of the integral). Therefore, the maximum value of  $x_i(\hat{t})$  holds when the negative term is equal to zero. In other words, it holds

$$\forall t : 0 < t \leq \hat{t} \rightarrow s_i(t) = 0$$

involving  $\hat{t} = T_i - C_i$ .  $\blacksquare$

Lemma 1 states that largest increasing variation of  $x_i(t)$  with respect to  $\bar{x}_i$  takes place when the state variable behaves as in Figure 1 (b). A similar result can be obtained for a decreasing variation, as stated in Lemma 2.

*Lemma 2:* The largest possible descending variation of the state variable  $x_i(t)$  with respect to  $\bar{x}_i$  is

$$\delta_i^{\text{dec}} = (\rho_i^{\text{out}} - \rho_i^{\text{in}}) C_i \quad (21)$$

*Proof:* The proof can be carried out as for Lemma 1.  $\blacksquare$

The behaviour local to one time period is related to global largest variations by Lemma 3.

*Lemma 3:* If  $U_i$  is assigned as in Equation 10, then:

$$\Delta_i^{\text{inc}} = \delta_i^{\text{inc}} \quad \text{and} \quad \Delta_i^{\text{dec}} = \delta_i^{\text{dec}} \quad (22)$$

Moreover, quantities in the Equation 22 are equal, so it can be defined:

$$\Delta_i \equiv \Delta_i^{\text{inc}} = \Delta_i^{\text{dec}} \quad (23)$$

*Proof:* Equation 22 is valid since, by Theorem 1, the initial condition is the same on each period. Therefore, results in Lemma 1 and Lemma 2, that are obtained for a generic period, have global validity over all the time  $t$ .

Equation 23 can be obtained by substitutions that involve Equations 5, 17, 21 and 22. ■

Theorem 2 allows to calculate the upper bound on the period  $T_i$  for a load  $\lambda_i$  such that, if used together with the load utilization  $U_i$  as in Equation 10, it guarantees that load  $\lambda_i$  will maintain its state variable  $x_i(t)$  within the required range  $[x_i^{\text{min}}, x_i^{\text{max}}]$ .

*Theorem 2:* If the period  $T_i$  is chosen in the interval  $(0, T_i^*]$ , where  $T_i^*$  is defined in (24),

$$T_i^* = \min \left\{ \frac{x_i^{\text{max}} - \bar{x}_i}{\rho_i^{\text{in}}(1 - U_i)}, \frac{\bar{x}_i - x_i^{\text{min}}}{\rho_i^{\text{in}}(1 - U_i)} \right\} \quad (24)$$

and  $U_i$  is assigned as in Equation 10, then

$$\forall t \in \mathbb{R} : t \geq 0 \rightarrow x_i^{\text{min}} \leq x_i(t) \leq x_i^{\text{max}}$$

*Proof:* Considering Lemma 3 and  $\bar{x}_i = x(0)$  as initial value, in order to keep the state variable into bounds, as stated in Equation 9, must be imposed:

$$\begin{cases} \bar{x} + \Delta_i \leq x_i^{\text{max}} \\ \bar{x} - \Delta_i \geq x_i^{\text{min}} \end{cases} \quad (25)$$

From Equations 5, 17, 23 and 25, it can be obtained by substitutions:

$$\begin{cases} T_i \leq \frac{x_i^{\text{max}} - \bar{x}}{\rho_i^{\text{in}}(1 - U_i)} \\ T_i \leq \frac{\bar{x} - x_i^{\text{min}}}{\rho_i^{\text{in}}(1 - U_i)} \end{cases} \quad (26)$$

Since both inequalities must hold,  $T_i$  is upper bounded by the minor of the two quantities in Equation 26, and (24) follows. ■

Since  $T_i^*$  represents an upper limit on the range where the load period could be selected, a shorter period could also be preferred, if needed, that still achieves the requirements on the state variable variation. It is worth to discuss the implications of such a possible choice. Shorter periods correspond to shorter distances in time between two consecutive request times. Therefore, shorter periods determine a more frequent activation of a load within a shorter time frame. This observation holds in general when the load is considered alone, i.e., it is not affected/preempted by the activations of other loads. In fact, in presence of more

than one load, preemptions may generate a similar effect, although in this case such behaviour does not emerge from the timing characteristics of a given load but arises from the interaction among load's activations. The effect is to narrow down the state variable variation range around  $\bar{x}_i$ . Although in general this behaviour may be considered as a desirable feature, a side effect needs to be taken into account, which is related with the characteristics of the physical process under control. Some types of loads, as high power electric motors, do not well tolerate sequences of activation/deactivation which are too close each other, since this may have a negative impact on the actuator's lifetime. Therefore, a larger state variable variation range (once the state variable is guaranteed to remain within the allowed range) can achieve a longer system lifetime.

## VI. EXAMPLE OF PHYSICAL SYSTEM

Given the system model and notation introduced in Section IV, in this section we provide an example of physical system having suitable characteristics to be represented using the proposed model (Figure 2). A vessel receives as input an amount of fluid with a constant flood capacity  $Q^{\text{in}}$  (e.g., expressed in  $[m^3/sec]$ ). The state variable is represented by the amount of fluid contained within the vessel. With adequate assumptions on the shape of the container, the fluid level  $h(t)$  can be used as system state variable.

The fluid level needs to be maintained within a predefined range  $[h^{\text{min}}, h^{\text{max}}]$  by acting on a hydro pump that, when active, pumps out a constant amount of fluid  $Q^{\text{out}}$  from the vessel, being  $Q^{\text{out}} > Q^{\text{in}}$ . The fluid level is not altered when the pump is inactive. The hydro pump is actuated by an electric motor that consumes a  $P_i$  amount of electric power when active, no power otherwise. The electric motor represents our  $\lambda_i$  load. If many of such systems are deployed for a given application, the goal is to achieve the application requirements while considering physical process behaviour and, on the other hand, to adequately schedule the electric motor activations to limit the peak load of power consumption.

## VII. APPLICATION OF THE PROPOSED TECHNIQUE: A SIMULATED EXAMPLE

In this section we present the application of the proposed modeling technique to a physical system composed by 3 electric devices having physical characteristics depicted in Table I. For the sake of simplicity, we limit this example to a set of loads having  $U < 1$ . This allows to clearly show how the absence of a proper management of load activations brings to the highest possible peak load in the worst case, while our approach improves (i.e., it decreases) the peak load. It is worth to note that, being  $U \leq 1$ , a real-time scheduling algorithm such as EDF is able to schedule the load set without any concurrent activation of loads. However, this fact does not limit the applicability

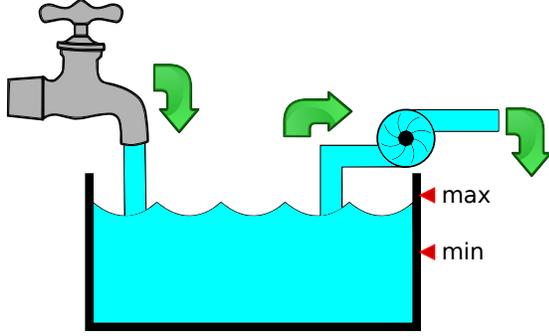


Figure 2. An hydraulic circuit where a vessel is filled at constant rate while an electric operated pump is used to maintain the fluid level within a pre-defined range.

Table I  
VALUES OF PHYSICAL PARAMETERS USED FOR THE SIMULATION

$i$	$\rho_i^{\text{in}}$	$\rho_i^{\text{out}}$	$P_i$	$\bar{x}_i$	$x_i^{\text{min}}$	$x_i^{\text{max}}$
1	1	4	1	3.4	2	4
2	1	5	2	4.2	1	5
3	2	4	3	4.0	3	5

of the proposed results since, when  $U > 1$ , loads can be partitioned into groups having  $U \leq 1$ , as shown in [18]. Although this solution may not bring to the optimal global schedule, it allows to use the proposed method to guarantee the limitation of the state variable.

Figure 3 shows the behavior of a simple on/off control technique applied to the considered load set. Each load is independently controlled so that the load is turned on when the state variable reaches the upper bound of the working range, and it is kept active until the lower bound is reached. This control strategy easily allows to individually maintain the state variable within the required working range. However, since the activation of loads is not coordinated, it is possible that more than one load is active at the same time, which turns out in an increase of the peak load of power consumption. In fact, several times in the depicted time range, the three considered loads are activated simultaneously, thus determining a peak load  $P = \sum P_i = 6$ .

The same three loads have been modeled and managed with the techniques introduced in this paper. Real-time parameters are calculated using the results presented in Section V, and they are reported in Table II. Figure 4 shows the behavior of state variables and the instantaneous consumed power  $p(t)$ . The figure shows that state variables are

Table II  
VALUES OF REAL-TIME PARAMETERS CALCULATED FOR THE SIMULATION

$i$	$U_i$	$T_i^*$	$C_i$
1	0.25	0.8	0.20
2	0.20	1.0	0.20
3	0.50	1.0	0.50

confined within the desired working ranges, while the load scheduling achieves to limit the peak load to  $P = \max_i P_i$ . In fact, in this example the load set is scheduled using the Earliest Deadline First (EDF) scheduling algorithm [13] and, since the total utilization is  $U = 0.95 \leq 1$ , the load scheduling guarantees that only one load is active at any given time. Notice that  $x_3(t)$  reaches its lower bound since the corresponding load is never preempted (it works in the worst condition specified by Lemma 2), while this is not the case for  $x_1(t)$  and  $x_2(t)$ .

Notice that in the proposed example we expressly generate a load set having  $U \leq 1$ . If  $U > 1$ , no scheduling algorithm would be able to achieve the activation of one load only at any given time. Therefore, concurrent activations of loads should have been adequately managed by either allowing concurrent activations or rejecting the activation of some loads to achieve the schedulability condition. The former approach resembles the real-time scheduling on multiprocessors, while the latter method involves techniques for overload management, widely studied in real-time computing systems. Both approaches are subject to ongoing research.

## VIII. CONCLUSIONS AND FUTURE WORKS

This paper presented a methodology for modeling the physical system of a cyber-physical energy system as periodic activities that can be scheduled by adapting traditional real-time scheduling algorithms. The goal of the proposed approach is to limit the peak of power consumption, which is a desirable feature for both the user and the energy provider, while achieving the requirements imposed by the application. In particular, in this paper we discussed the application to load sets where loads have linear dynamics behaviours.

The proposed approach represents a pioneering approach to the use of real-time scheduling techniques to organize the activation of electric loads in a cyber-physical energy system. In this paper, a number of simplifying assumptions have been made, such as considering periodic activations only, linear state variable's dynamics, etc., thus future works will address the relaxation of such restrictive assumptions.

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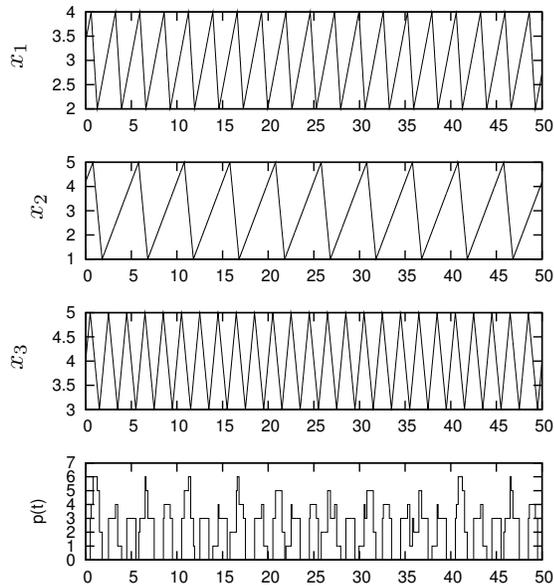


Figure 3. Example of a simple on/off control strategy using the load parameters specified in Table I.

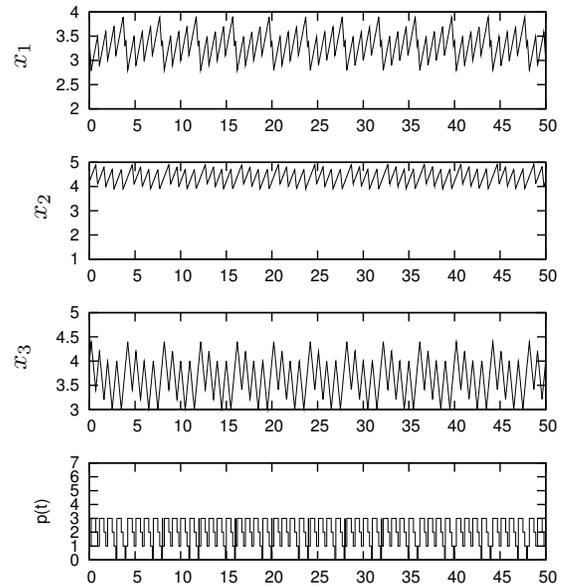


Figure 4. Example of schedule generated by the EDF scheduling algorithm using the load parameters specified in Table I.

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